On model selection, validation and reconstruction in the context of physical cosmology

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Korea Astronomy and Space Science Institute (KASI) University of Science and Technology (UST) The 2nd Shanghai Assembly on Cosmology and Structure Fromation 30 October-3 November 2023,

SJTU-TDLI, Shanghai

Era of Precision Cosmology

We try to reconstruct and understand the dynamics of the universe and properties of its constituents using various measurements and statistical techniques. Phenomenological and then theoretical works can follow to place constraints on suggested models and their parameters.

Baryon density

Dark Matter: density and characteristics

FLRW?

Neutrino species, mass and radiation density

Dark Energy: density, model and parameters

Initial Conditions: Form of the Primordial Spectrum and Model of Inflation and its Parameters

Epoch of reionization

Curvature of the Universe Hubble Parameter and the Rate of Expansion

What do we do?

- There are various reconstruction approaches, parametric and non-parametric.
- There have been many phenomenological and theoretical models proposed (recently, to alleviate tensions).

Reconstruction \rightarrow Phenomenology \rightarrow Theory

- There have been continuous attempts looking for systematics in various data.
- These models/reconstructions can be very different. **How do we compare them?**

Consistency of a proposed model and the data:

Frequentist Approach:

Assuming a proposed model, the probability of the observed data must not be insignificant. Best is to do large number of careful simulations based on a well defined covariance error-matrix.

Bayesian Approach:

Priors and simplicity of the proposed model *also* matters (in model comparison)

Chi square analysis plays a crucial role in calculation of the likelihood in both approaches

Why things are more complicated than what we think…

Likelihood

We are interested to calculate the probability of the observed data given the model.

$$
\chi^{2} = \sum_{i}^{N} (\mu_{i}^{t} - \mu_{i}^{e})^{T} Cov^{-1}(\mu_{i}^{t} - \mu_{i}^{e})
$$

$$
P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}
$$

$$
Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.
$$

When data is uncorrelated

What if the exact form of the error matrix is not known? **Point 1**

$$
\chi^{2} = \sum_{i}^{N} (\mu_{i}^{t} - \mu_{i}^{e})^{T} Cov^{-1}(\mu_{i}^{t} - \mu_{i}^{e})
$$

$$
\sigma_i^2 = \sigma_{i(data)}^2 + \sigma_{(sys)}^2
$$

$$
P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}
$$

e.g. *The case of Type Ia supernovae*

$$
Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.
$$

This can still happen!

 $\chi^2 = \sum_{i} (\mu_i^t - \mu_i^e)^T Cov^{-1}$ *i N* $\sum (\mu_i^t - \mu_i^e)^T Cov^{-1}(\mu_i^t - \mu_i^e)$

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Pantheon+ data Keeley, Shafieloo, L'Huillier, arXiv:2212.07917

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This can still happen!

Point 1

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Pantheon+ data Keeley, Shafieloo, L'Huillier, arXiv:2212.07917

 $Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.$

Likelihood and Model Fitting

When number of data points is more than ~30 one can use relative chi square for likelihood analysis and N, number of free parameters of the fitting function, will become the degrees of freedom.

$$
\chi^{2} = \sum_{i}^{N} (\mu_{i}^{t} - \mu_{i}^{e})^{T} Cov^{-1}(\mu_{i}^{t} - \mu_{i}^{e})
$$

In likelihood estimation:

$$
\chi^2 \longrightarrow \Delta \chi^2
$$

$$
\Delta \chi^2 = \chi^2 - \chi^2_{best}
$$

$$
P(\chi^2; N) = \frac{2^{-N/2}}{\Gamma(N/2)} \chi^{N-2} e^{-\chi^2/2}
$$

$$
Prob(\chi^2; N) = \int_{\chi^2}^{\infty} P(\chi^2; N) d\chi'^2.
$$

Bayesian Analysis

- Bayesian approach provides the means to incorporate *prior* knowledge in data analysis.
- Bayes's law states that the posterior probability is proportional to the product of the likelihood and the prior probability.

Posterior probability and the priors:

Likelihood

Prior probability

$$
p(x|d) = \frac{p(d|x)p(x)}{p(d)}
$$

Posterior probability

Normalization factor

Model fitting has Bayesian essence since *assume that we are considering a correct model.* **Point 2**

Bayesian Evidence and Model Selection

Bayesian evidence: Integral of (likelihood)x(prior) over \bullet the parameter space: $Z = \int L(\theta) \pi(\theta) d\theta$

Bayes factor: Ratio of the evidence of the two models: \bullet Δ logZ = logZ(M₁) - logZ(M₂)

> 1 2

Supports Model 1 over Model 2 when Δ logZ have a positive value

Bayesian Evidence and Model Selection

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Supports Model 1 over Model 2 when Δ logZ have a positive value

Point 3

1 **How reliable are**

these scales?

3

Standard Model of Cosmology

Universe is Flat Universe is Isotropic Universe is Homogeneous Dark Energy is Lambda (w=-1) Power-Law primordial spectrum (n_s=const) Dark Matter is cold All within framework of FLRW Let's solve Hubble tension with evolving DE!

Phenomenologically Emergent Dark Energy (PEDE)

No Dark Energy in the past and it acts as an emergent phenomena:

Allows lower rate of expansion in the past and higher rate of expansion at late times

$$
\Omega_{\rm DE}(z)\,=\,\Omega_{\rm DE,0}\times[1-\tanh{(\log_{10}(1+z))}]
$$

$$
w(z) = -\frac{1}{3\ln 10} \times \frac{1 - \tanh^2 [\log_{10}(1+z)]}{1 - \tanh [\log_{10}(1+z)]} - 1
$$

$$
= -\frac{1}{3\ln 10} \times (1 + \tanh [\log_{10}(1+z)]) - 1.
$$

Li and Shafieloo, ApJ Lett 2019

Generalized Emergent Dark Energy (GEDE)

$$
\widetilde{\Omega}_{\rm DE}(z) = \Omega_{\rm DE,0} \frac{1 - \tanh\left(\Delta \times \log_{10}\left(\frac{1+z}{1+z_t}\right)\right)}{1 + \tanh\left(\Delta \times \log_{10}\left(1+z_t\right)\right)}
$$

-Has one degree of freedom for DE sector

$$
w(z) = -\frac{\Delta}{3\ln 10} \times \left(1 + \tanh\left(\Delta \times \log_{10}\left(\frac{1+z}{1+z_t}\right)\right)\right) - 1.
$$

6 $\Delta = -1$ $\Delta = 0$ (ACDM) 5 $\Delta = 1$ (PEDE) $\Delta = 10$ $\overline{4}$ 3 $\vec{\mathcal{N}}$ $\overline{2}$ $\mathbf{1}$ $\mathbf 0$ -1 0.4 0.6 0.2 0.8 0.0 1.0 $\Omega_{m,0}$

$$
\Omega_{\rm DE}(z_t) = \Omega_{m,0} (1+z_t)^3
$$

-LCDM and PEDE are both included at special limits

$$
\Delta = 0
$$

$$
\Delta=1^-
$$

PEDE

LCDM

Li and Shafieloo, ApJ 2020 (arXiv:2001.05103)

Generalized Emergent Dark Energy (GEDE)

Full analysis using various combination of the data

Model Comparison: Bayesian evidence analysis in strong support of emergent dark energy

Generalized Emergent Dark Energy (GEDE)

Full analysis using various combination of the data

Current tensions allow us to find models statistically better (?) than LCDM but are all tensions resolved?

Model Comparison: Bayesian evidence analysis in strong support of emergent dark energy

No!

True for any successful evolving DE model!

W. Yang,et al, PRD 2021 [arXiv:2103.03815]

Distribution of Bayesian Evidence:

• Be cautious about Jeffery's scale!

Distribution of Bayes factors can greatly depend on the models and the data!

On The Distribution of Bayesian Evidences

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ABSTRACT

We look at the distribution of the Bayesian evidence for mock realizations of supernova and baryon acoustic oscillation data. The ratios of Bayesian evidences of different models are often used to perform model selection. The significance of these Bayes factors are then interpreted using scales such as the Jeffreys or Kass & Raftery scale. First, we demonstrate how to use the evidence itself to validate the model, that is to say how well a model fits the data, regardless of how well other models perform. The basic idea is that if, for some real dataset a model's evidence lies outside the distribution of evidences that result when the same fiducial model that generates the datasets is used for the analysis, then the model in question is robustly ruled out. Further, we show how to assess the significance of a hypothetically computed Bayes factor. We show that the range of the distribution of Bayes factors can greatly depend on the models in question and also the number of data points in the dataset. Thus, we have demonstrated that the significance of Bayes factors needs to be calculated for each unique dataset.

Key words: dark energy - cosmological parameters - methods: statistical

Keeley and Shafieloo, MNRAS 2022

Data with OK quality

Data with worse quality

Bayes Factor:

• Be cautious about Jeffery's scale!

Point 3

Distribution of Bayes factors can greatly depend on the models and the data!

 0.5 Data: ACDM, Models: ACDM-TDE Data: ACDM, Models: ACDM-PEDE Data: ACDM, Models: PEDE-TDE 0.4 Data: TDE Models: TDE-ACDM Data: TDE, Models: ACDM-PEDE Data: TDE Models: TDE-PEDE Data: PEDE, Models: ACDM-TDE 0.3 Data: PEDE, Models: PEDE-ACDM Data: PEDE, Models: PEDE-TDE 0.2 $0.1\,$ $0.0 -10$ -5 10 15 20 25 \triangle logZ

Data with worse quality

Keeley and Shafieloo, MNRAS 2022

See also:

Starkman et al, arXiv:0811.2415 Jenkins & Peacock, MNRAS 2011; Nesseris & Garcia-Bellido, JCAP 2013; Joachimi et al., A&A 2021;

Data with OK quality

Model Validation

Bayesian evidence approach is solid but only can find the better model among the candidates (or less wrong model/ranking models)

Importance of Model Validation

Conventional Bayesian Evidence Approach

Both models are wrong! Shafieloo, Keeley, Shafieloo, L'Huillier, JCAP 2022 **Point 4**

 \rightarrow When true model is unknown, finding a statistical anchor is not trivial

 \rightarrow One can attempt using reliable non-parametric reconstructions

Iterative Smoothing Method

- The non-parametric method to reconstruct the distance modulus and expansion history of the universe Shafieloo et al. 2006, 2018; Shafieloo. 2007; Shafieloo & Clarkson 2010
- Starts from initial guess of distance modulus, but generates model-independent reconstruction of distance modulus with lower χ^2 value after numerous iterations

$$
\hat{\mu}_{n+1}(z) = \hat{\mu}_n(z) + \frac{\delta \mu_n^T \cdot c^{-1} \cdot W(z)}{1^T \cdot c^{-1} \cdot W(z)}
$$
 (C: Covariance matrix of the data)

$$
\mathbf{1}^T = (1, \dots, 1), \mathbf{W}_i(z) = \exp\left(-\frac{\ln^2(\frac{1+z}{1+z_i})}{2\Delta^2}\right), \delta \mu_n|_i = \mu_i - \hat{\mu}_n(z_i) \text{ (}\Delta \text{: Smoothing width)}
$$

$$
\chi_n^2 = \delta \mu_n^T \cdot C^{-1} \cdot \delta \mu_n
$$

- Derive the likelihood distribution function $P(\Delta \chi^2)$ (for a large number of data realizations), where $\Delta \chi^2 = \chi^2_{\rm smooth} - \chi^2_{\rm best-fit}$, when the true model is assumed Koo et al. 2021, JCAP, 03, 034
- \cdot $\chi^2_{\rm smooth}$: χ^2 of the converged reconstruction using smoothing method
- $\chi^2_{\text{best-fit}}$: Best-fit χ^2 of the correct model fits the data

Testing Models based on Likelihood Distribution

- \cdot $P(\Delta \chi^2)$ have no dependence on the true model and depends only on the covariance matrix of the data \rightarrow One $\Delta \chi^2$ for given confidence (Ruler) Koo et al. 2021, JCAP, 03, 034
- The model being tested is ruled out if the \bullet $\Delta \chi^2$ value is lower than the ruler

• Likelihood distributions exclude both models

Non-parametric reconstruction and Model Validation

Model Validation

Bayesian evidence approach is solid but only can find the better model among the candidates (or less wrong model/ranking models)

Importance of Model Validation

One can design robust statistical approaches for model validation

Point 4

Conventional Bayesian Evidence Approach

Both models are wrong!

Iterative smoothing validation approach

> Koo, Keeley, Shafieloo, L'Huillier, JCAP 2022

Application of model validation

Ruling Out New Physics at Low Redshift as a solution to the H0 Tension

Keeley and Shafieloo, Phys. Rev. Lett, 2023

Exploring an **extensive** physical space with Crossing functions for validation (Chebyshev polynomials)

pplication of model validation

Ruling Out New Physics at Low Redshift as a solution to the H0 Tension

Even in such extensive physical space, inference on H0 is not consistent with SH0ES.

Keeley and Shafieloo, Phys. Rev. Lett, 2023

Application of model validation

Isn't it suspicious that nothing works?!

maybe there are some systematics somewhere?

66

68

 H_0

 0.30

 0.32

 0.34

 0.28

Even in such extensive physical space, inference on H0 is not consistent with SH0ES.

 70

 72

Validation of a large number of models can hints towards systematic

$(P$ resent) Lets talk about tensions again…

Standard Model of Cosmology

Universe is Flat Universe is Isotropic Universe is Homogeneous Dark Energy is Lambda (w=-1) Power-Law primordial spectrum (n_s=const) Dark Matter is cold All within framework of FLRW On Importance of nonparametric reconstruction **When we don't know what to look for! Point 5**

Model Independent Reconstruction of Primordial Spectrum *Let's Reconstruction Leads the way!* **Point 5**

Figure 4. Reconstruction of the shape of the primordial power spectrum in 16 bands after marginalising over the Hubble constant, baryon and dark matter densities, and the redshift of reionization.

Beyond Power-Law: there are some other models consistent to the data.

Beyond Power-Law: *there are some other models consistent to the data.*

Whipped Inflation

Hazra, Shafieloo, Smoot, JCAP 2013 Hazra, Shafieloo, Smoot, Starobinsky, JCAP 2014A Hazra, Shafieloo, Smoot, Starobinsky, JCAP 2014B Hazra, Shafieloo, Smoot, Starobinsky, Phys. Rev. Lett 2014 Hazra, Shafieloo, Smoot, Starobinsky, JCAP 2016 Hazra et al, JCAP 2018 Debono, Hazra, Shafieloo, Smoot, Starobinsky, MNRAS 2020 Hazra, Paoletti, Debono, Shafieloo, Smoot, Starobinsky, JCAP 2021

Forms of PPS and Effects on the Background Cosmology

- Flat Lambda Cold Dark Matter Universe (LCDM) with power–law form of the primordial spectrum
- It has 6 main parameters.

 $|C_i = \sum G(l,k)P(k)|$

 C_l

obs

3

 $P(k) = A_s[\frac{k}{k}]^{n_s-1}$

2

2

 $G($ l,

1

 Ω_h

 Ω_m

 H_\circ

 $A_{\overline{s}}$

 $n_{\overline{s}}$

1

Forms of PPS and Effects on the Background Cosmology

• Cosmological parameter estimation with free form primordial power spectrum

2

G(I,

 $P(k)$

3

3

2

 $C_l = \sum G(l,k)P_l$

 C_l

4

obs

 Ω_h Ω_m H_{α}

 \overline{S}

1

Hazra, Shafieloo, Souradeep, JCAP 2019 Keeley et al, MNRAS 2020

Background Cosmological Parameters and PPS

We use the reconstructed PPS for parameter estimation, similar to what we do with PL.

One spectrum to cure them all: looking for signature from early Universe to solve major anomalies and tensions in cosmology

Curvature and A_lens anomalies

Hazra, Antony, Shafieloo : JCAP 2022

One spectrum to cure them all: Signature from early Universe solves major anomalies and tensions in cosmology

Addressing Majour Anomalies and tensions

Hazra, Antony, Shafieloo :JCAP 2022

Now we know what to look for! **Point 5**

Reconstruction à **Phenomenology** à**Theory**

i, Hazra, Shafieloo, P

Current Status

Open problem. Many tensions and hints for various **systematics**

Many theoretical/phenomenological models are proposed to ease the tensions. None is convincing so far (none can pass all validation tests).

Not possible to resolve all problems with minimal modification of the standard model. This has helped the standard model to survive so far.

Model independent consistency test between various data is essential to rule out systematics.

Point 6

Looking for systematics

Model independent consistency test between various data is essential to rule out systematics.

GP for Falsification Shafieloo, Kim, Linder, PRD 2012 Shafieloo, Kim, Linder, PRD 2013 Hwang et al, JCAP 2023

Consistency of SDSS BAO and Pantheon SN Ia data Keeley, Shafieloo, Zhao,…, MNRAS 2021 [arXiv:2010.03234] [SDSS IV paper]

 $H0rd = 10040 \pm 140$ km/s and $\Omega k = 0.02 \pm 0.20$

Point 6

Future Perspective

High possibilities for systematics in different data

Need for independent measurements

Two key questions:

Power-Law Primordial Power Spectrum? Lambda Dark Energy?

Tip of the Red Giant Branch

Future **Perspective**

Figure 17. A plot of H_0 values as a function of time. The points and shaded region in black are those determined from measurements of the CMB; those in blue are Cepheid calibrations of the local value of H_0 ; and the red points are TRGB calibrations. The red star is the best-fit value obtained in this paper. Error bars are 1σ .

Figure 18. Completely independent calibrations of H_0 . Shown in red is the probability density function based on our LMC CCHP TRGB calibration of CSP-I SNe Ia; in blue is the Cepheid calibration of H_0 (Riess et al. 2016), using the Milky Way parallaxes and the maser distance to NGC 4258 as anchors (excluding the LMC). The Planck value of H_0 is shown in black.

Cosmology with Strong Lens Systems: Has become already competative!

H0 from Strongly Lensed systems

$$
H_0 = 72.8^{+1.6}_{-1.7} \text{ km/s/Mpc}
$$

2.3% model-independent measurment of Hubble constant

Liao, Shafieloo, Keeley, Linder, ApJ Letters 2020

Liao, Shafieloo, Keeley, Linder, ApJ Letters 2019

H0LiCOW I. H0 Lenses in COSMOGRAIL's Wellspring

Suyu et al. MNRAS 2017

Future perspective (late universe, SN Ia)

Scolnic,et al, arXiv:1903.05128

Future perspective (late universe; BAO & RSD)

Aghamousa et al, [arXiv:1611.00036] DESI Collaboration

Future perspective (late universe; BAO, RSD)

DESI Y1 data will be released s and it will be **better than all ex LSS data combined**

arXiv:2306.06307 DESI SV arXiv:2306.06308 DESI EDR

Aghamousa et al, [arXiv:1611.00036] DESI **Collaboration**

Astro2020 Future perspective [G-Waves and Standard Sirens]

Figure 1: Hubble constant uncertainty (1σ) as a function of combined GW events with associated EM counterpart. The shaded regions show the impact of the peculiar velocity uncertainty between 100 and 400 $km s^{-1}$ for different distance reaches D_{*} . The latest results from standard candles (SH0ES, $\overline{[13]}$) and CMB $\frac{1}{100}$ (*Planck*, [14]) are also shown.

Palmese et al, arXiv:1903.04730

Future **Perspective** (primordial)

Full picture

Complete reconstruction analysis with polarization data

$$
C_{\ell}^{TT} = \int \frac{dk}{k} P(k) G_{\ell}^{TT}(k)
$$

$$
C_{\ell}^{EE} = \int \frac{dk}{k} P(k) G_{\ell}^{EE}(k)
$$

$$
C_{\ell}^{BB} = \int \frac{dk}{k} P(k) G_{\ell}^{BB}(k)
$$

$$
C_{\ell}^{TE} = \int \frac{dk}{k} P(k) G_{\ell}^{TE}(k)
$$

Searching for correlations!

$$
\left|P_{_{\textup{S}}}(k),P_{_{\textup{T}}}(k),P_{_{\textup{iso}}}(k)\right|
$$

Primordial power spectra from Early universe

Post recombination Radiative transport kernels in a given cosmology

 $\int G^{EE}(k) \; G^{BB}$

Features with Future of CMB (S4)

With Cosmic Origins Explorer (CORE)-like survey specification

- Large scale suppressions can not be detected with high significance
- Some of the intermediate and small scale oscillations can be detected, if present

From 2D to 3D Future Perspective

Using LSS data to test early universe scenarios

- 1. We need to estimate matter power spectrum but we observe galaxies. Hence we have to model linear clustering bias and estimate its parameters accurately and precisely to connect the observables to theory. Bias modeling would be different for different surveys and susceptible to systematics.
- 1. Does power spectrum (or bi-spectrum, etc) necessarily contains all the information in 3D data of LSS? Can't reducing dimensionality of the data wash out some information?

Data will be hugely better…but we have to be careful!

Cosmology vs Systematics

- With higher quality of the data the role of systematics will become more and more prominent.
- Higher precision may cost us uncontrollable bias if we make wrong assumptions. **Point 6**

What we should be worried about!

Conclusion

- Many statistical tools are not used appropriately in cosmology and astrophysics and results can be strong but invalid conclusions.
- HO tension (and some others) seems remaining persistent in the context of the LCDM model. This can open ways for competitive alternatives (GEDE?, EDE, features in PPS?) but we should not over-sell these models.
- *Tensions are not resolved with minimal extensions of the standard model and there is no clear resolution.* It is highly possible, from statistical point of view, that there are *systematics* in some of the data and we might need *new physics* too. *It can be a combination of both!* New independent measurements and observations can help to clear things up.
- With higher quality data, the effect of systematics and wrong assumptions
are much more prominent in introducing substantial inaccuracies. This is a are much more prominent in introducing real challenge to avoid making big fake discoveries.

Conclusion

- Standard Model of Cosmology fits different data pretty well *individually* but there are tensions fitting different combinations of the data.
- HO tension (and some others) seems remaining persistent in the context of the LCDM model. This can open ways for competitive alternatives *(GEDE?, EDE, features in PPS?)*.
- *Tensions are not resolved with minimal extensions of the standard model and there is no low redshift resolution.* It is highly possible that there are *systematics* in some of the data and we might need *new physics* too. *It can be a combination of both!* New independent measurements and observations can help to clear things up.
- **First target can be testing different aspects of the standard model. If it is not '***Lambda***' dark energy or '***power-law'* **primordial spectrum** then we can look further. It is possible to focus the power of the data for the purpose of the falsification. *Next generation of astronomical observations*, (DESI, Euclid, LSST, WFIRST, SKA(?), etc) will make it much more clear about the status of the concordance model in 2020s.