

*application of constrained  
simulations in understanding  
the galaxy-halo connection:  
the case for halo assembly bias*

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Yen-Ting Lin  
*ASIAA*

Hironao Miyatake, Hong Guo, Yi-Kuan Chiang, Kai-Feng Chen,  
Ting-Wen Lan, Yu-Yen Chang

*with special thanks to*

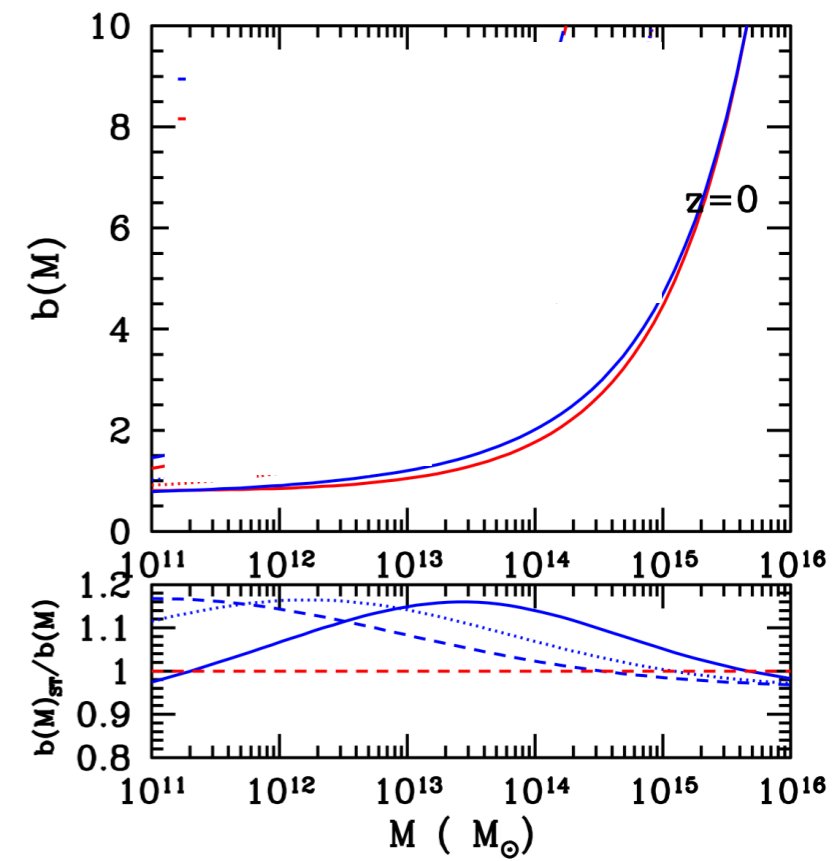
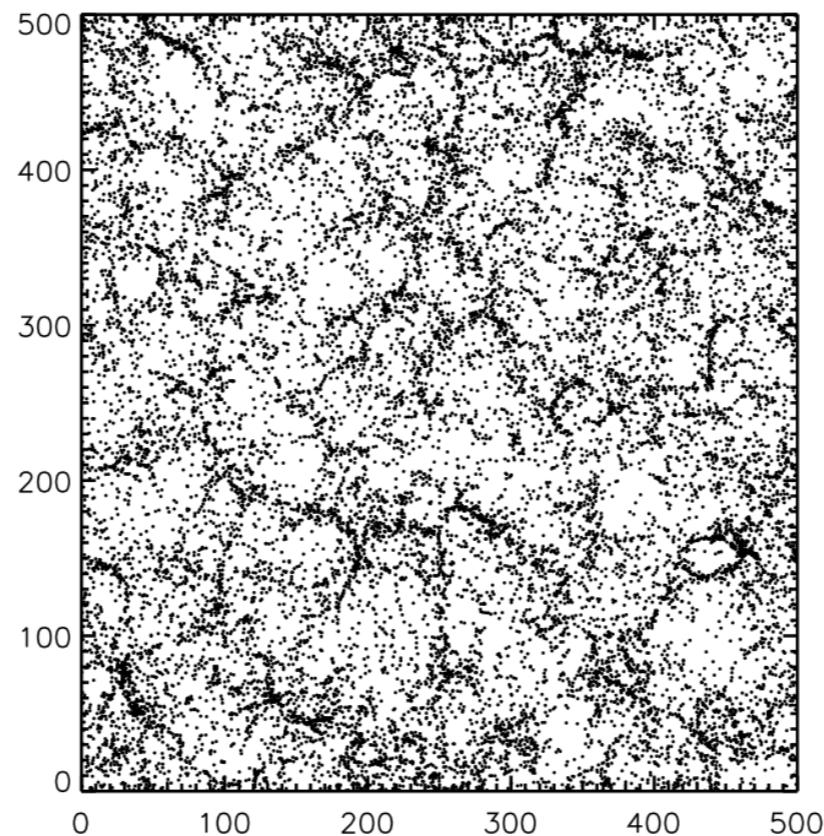
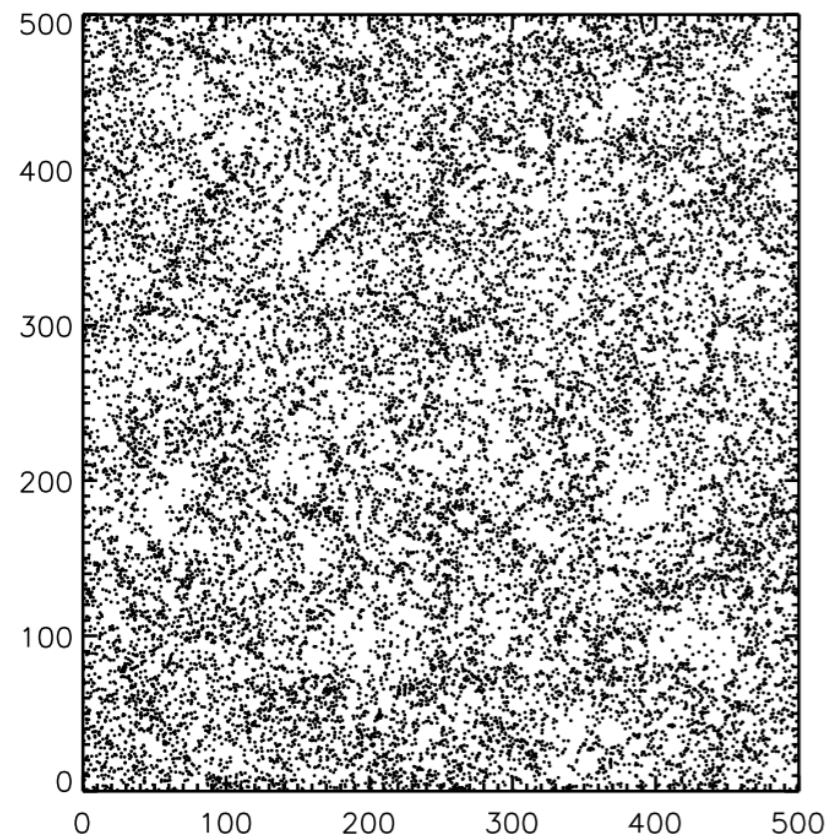
Huiyuan Wang, Xiaohu Yang, & Houjun Mo



# assembly bias?

- dark matter halos biased tracers of matter, with bias primarily as a function of halo mass  $\Rightarrow$  more massive halos are more biased
- a secondary effect is *assembly bias*: bias also depends on the halo formation time
  - for low mass halos ( $\sim 10^{12} h^{-1} M_{\odot}$ ), those that form earlier would cluster more strongly (having  $\sim 40\%$  larger bias)
  - for cluster-scale halos, youngest halos are  $\sim 10\%$  more biased than oldest ones

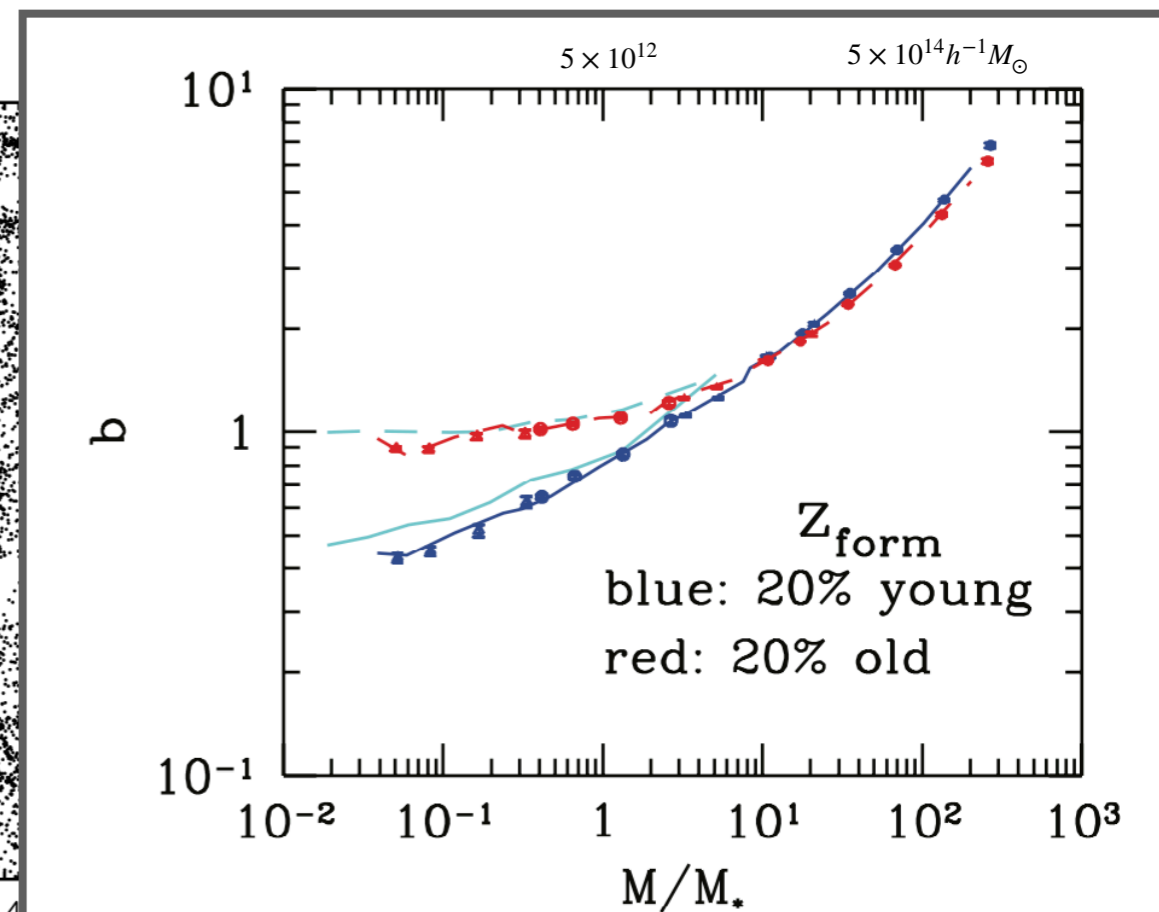
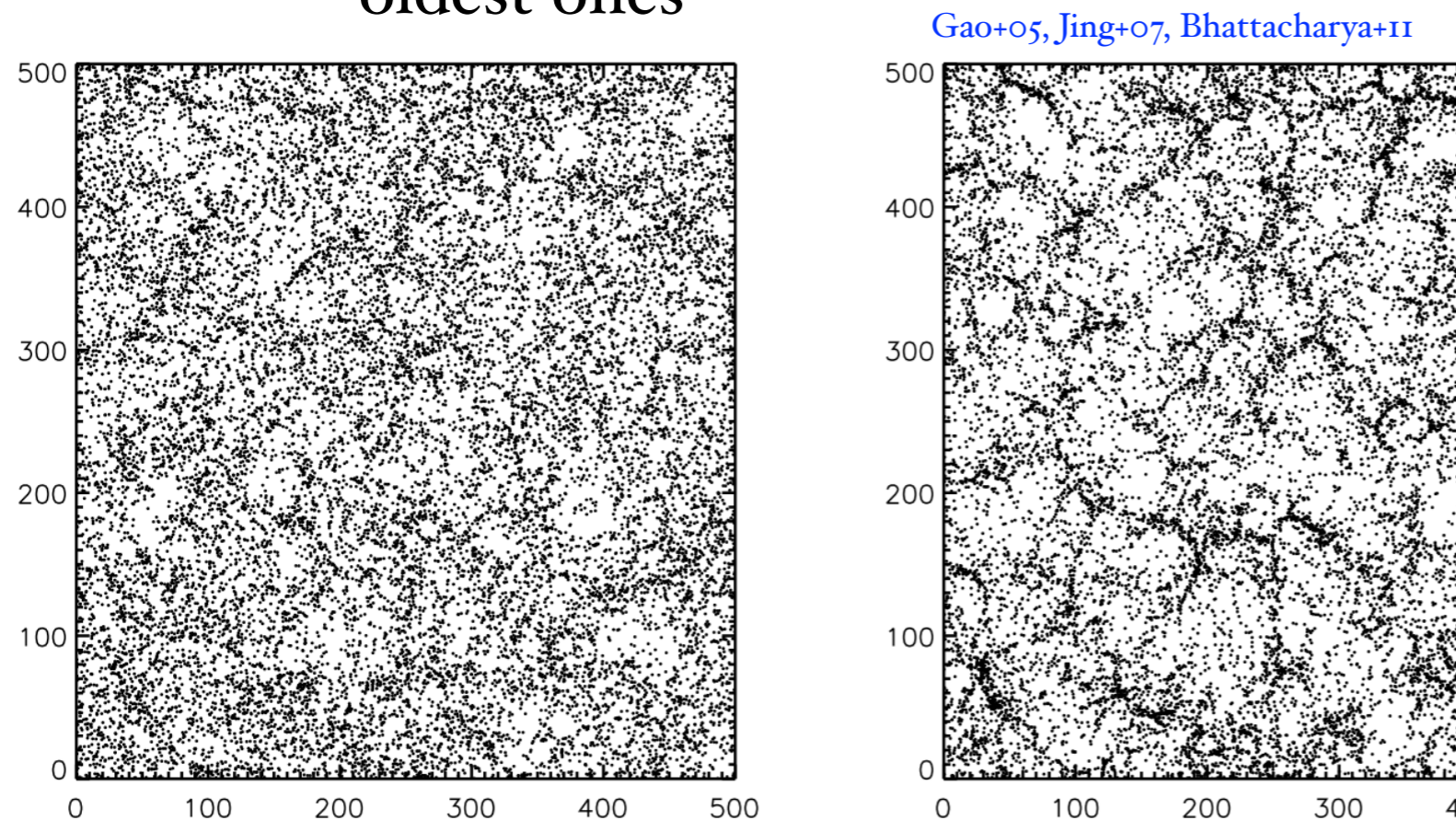
Gao+05, Jing+07, Bhattacharya+11





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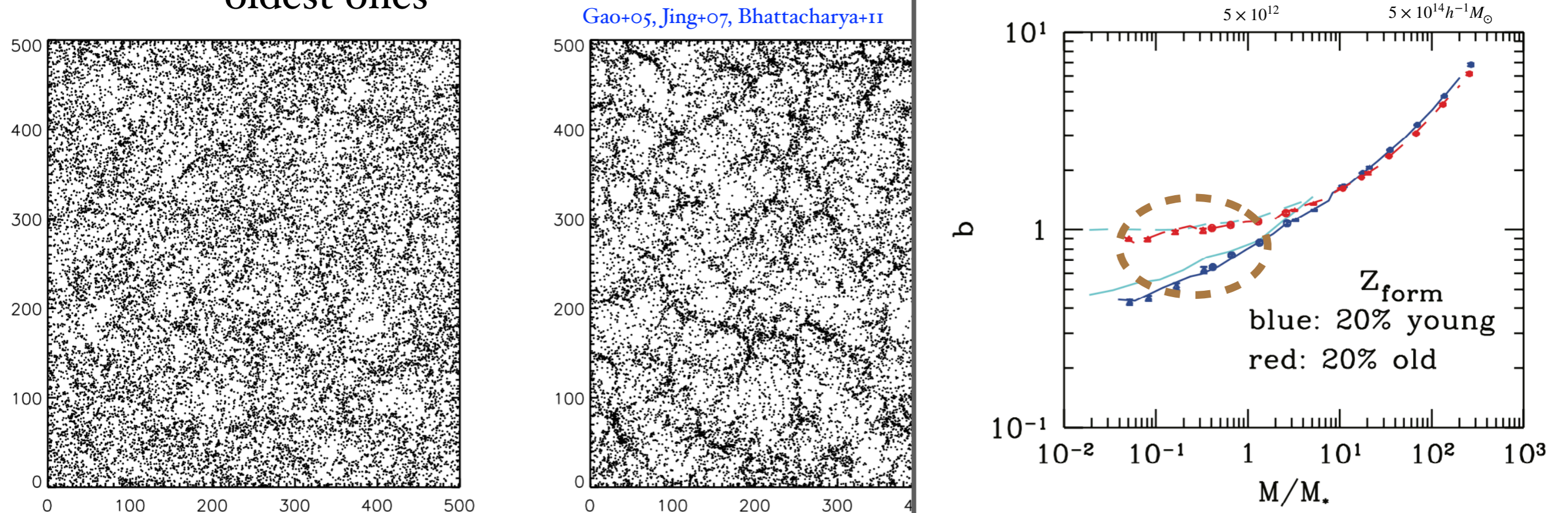
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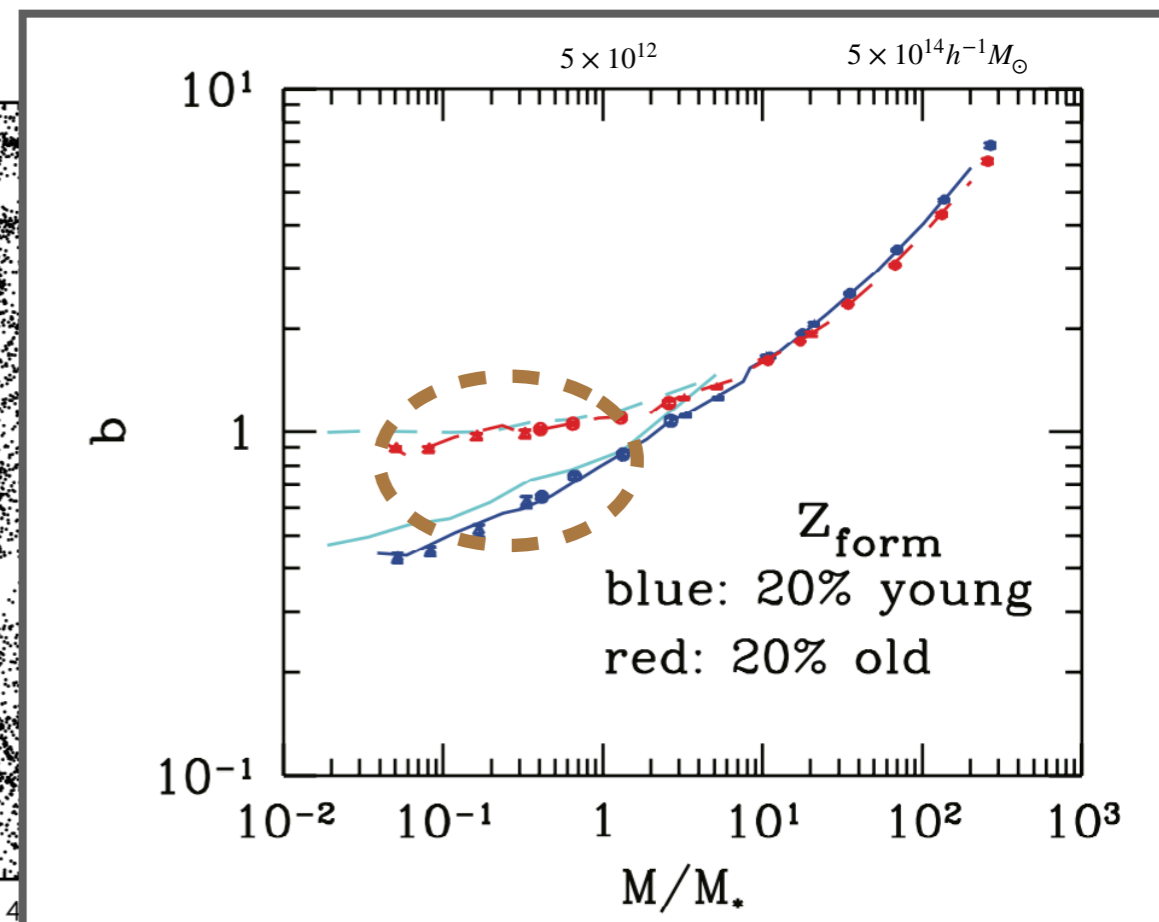
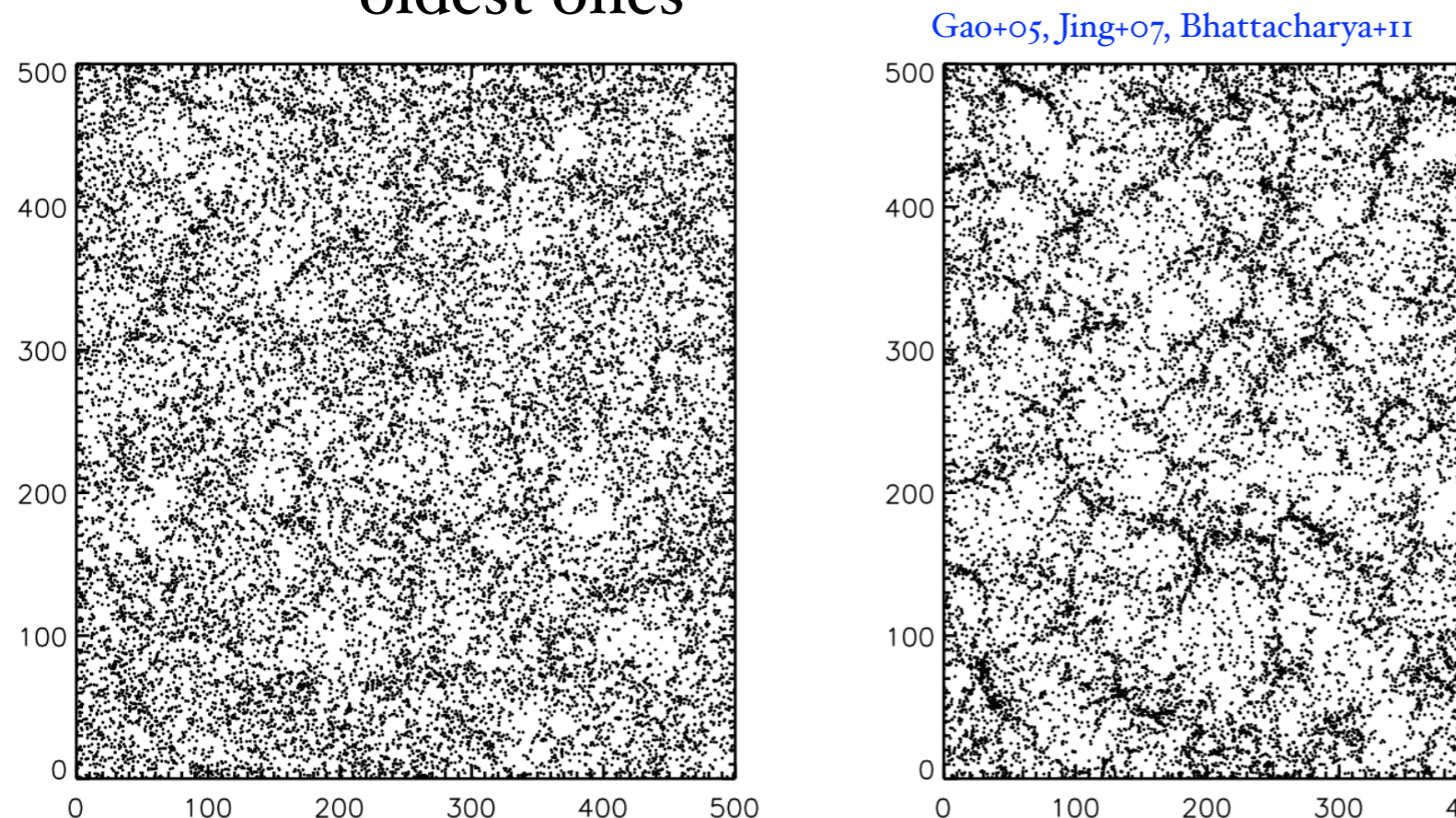




# assembly bias?

as assembly bias is a robust prediction/feature of  $\Lambda$ CDM  
it is important to find observational evidence for it!!

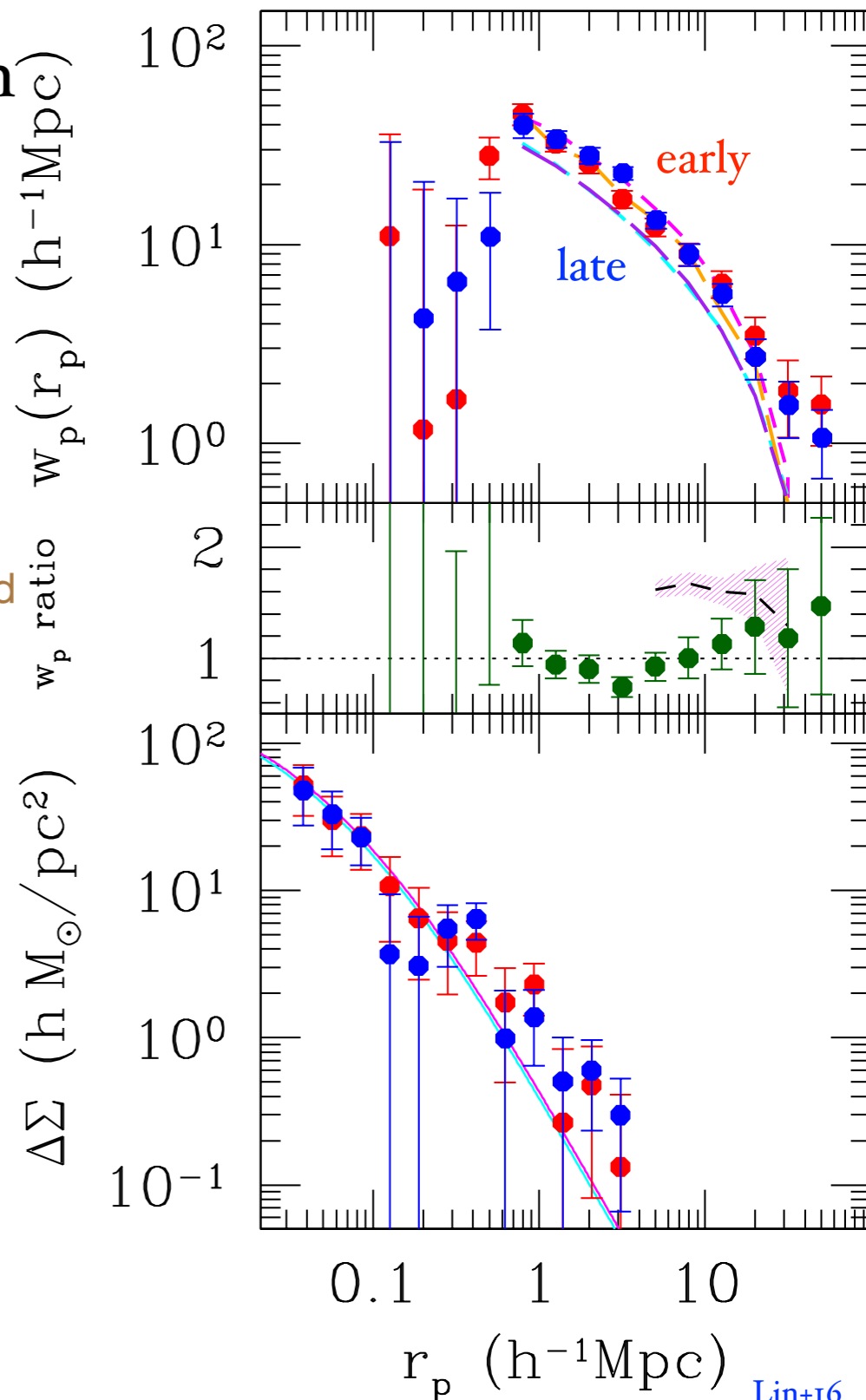
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# non-detection of assembly bias

- we have constructed a pair of early- and late-forming halos, selected by the star formation history (SFH) of the central galaxy
  - assuming SFH of central galaxy correlates well with the formation history of the halo
- masses are  $(9 \pm 2) \times 10^{11} h^{-1} M_{\odot}$  and  $(8 \pm 2) \times 10^{11} h^{-1} M_{\odot}$
- theoretical expectation derived from N-body simulations, taking into account uncertainties in halo mass distribution
  - log-normal form assumed
  - probable values of centroid & width allowed by measured lensing signal
- probability for theory to be consistent with observation is  $5 \times 10^{-5}$

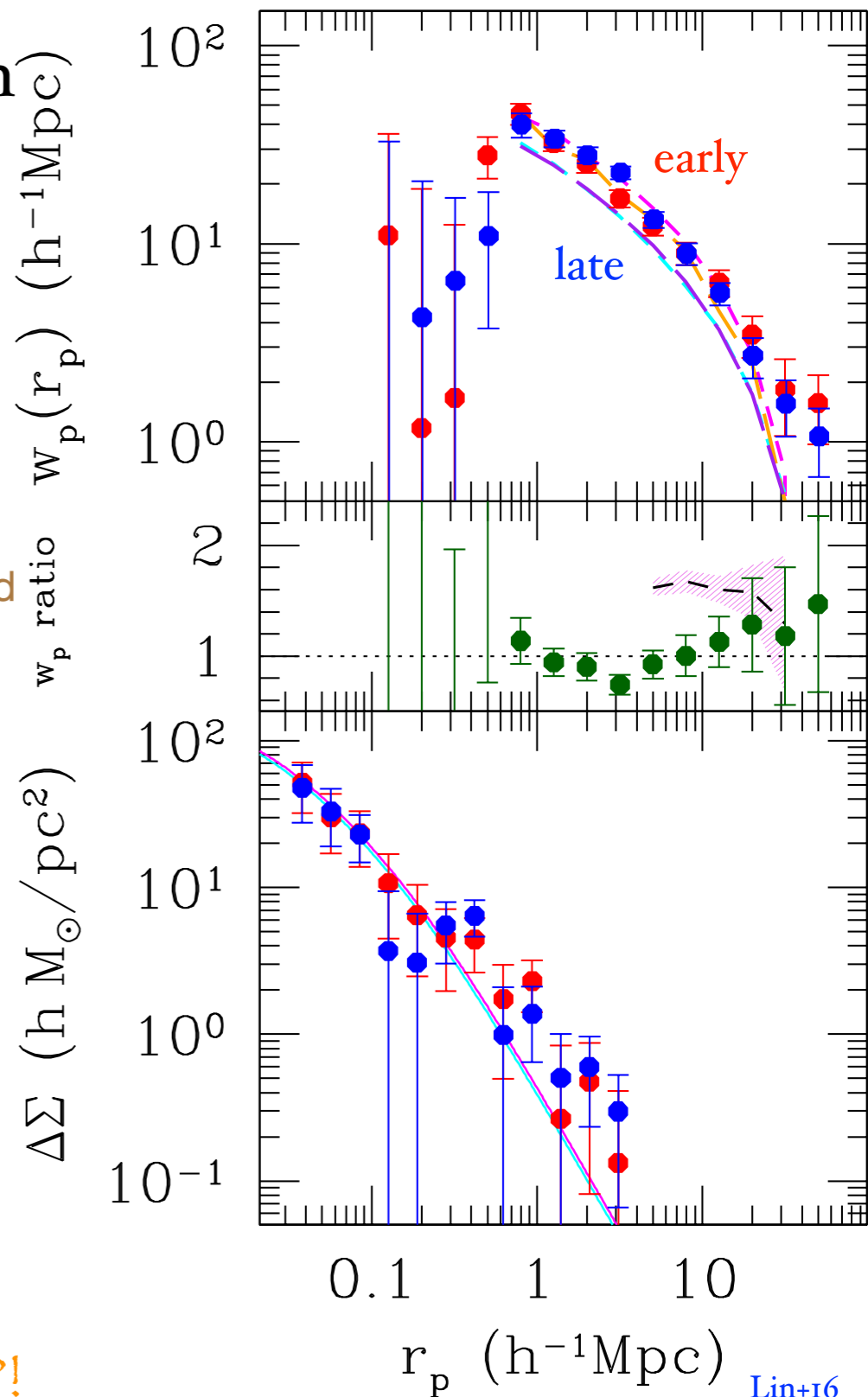
early-to-late bias ratio squared



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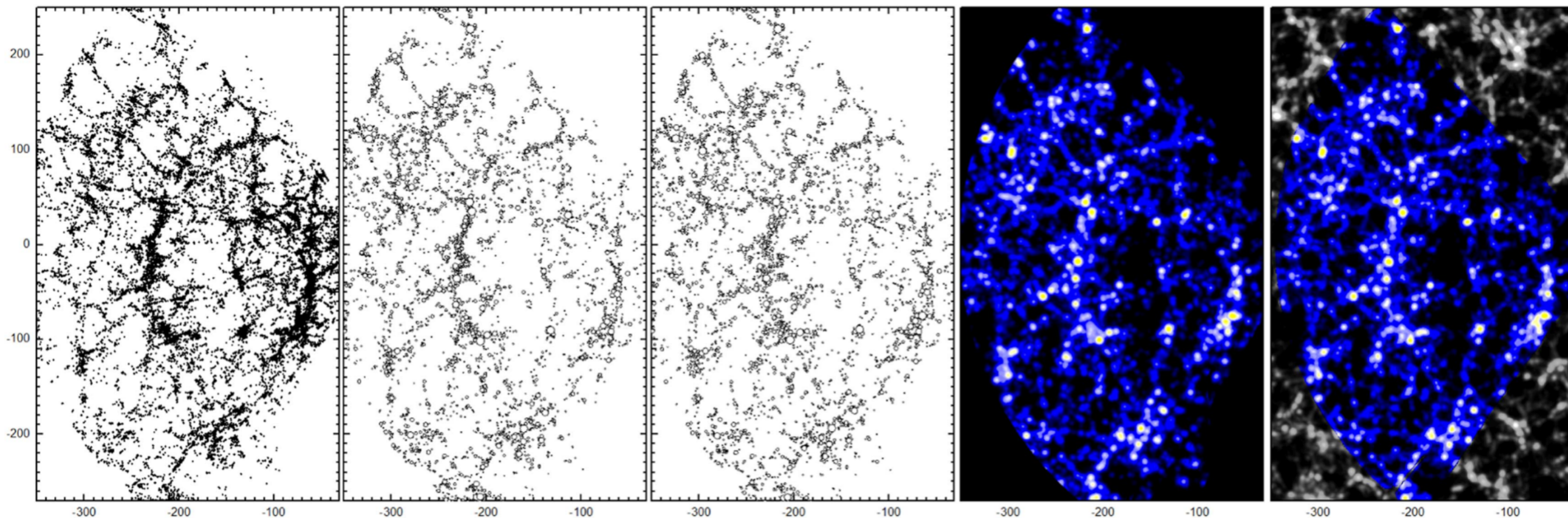
insufficient data quality?!



# *Elucid* to the rescue

- hard-learned lesson: *reliable* proxy of halo formation time working on *individual* halo basis
- it would be a dream come true if we have the mass accretion history (MAH) of the clusters!
- using the group catalog of Yang et al. (2007), Wang et al. (2016) have run a constrained simulation of the local Universe (SDSS DR7,  $z < 0.12$ ) called *Elucid*

Wang+16





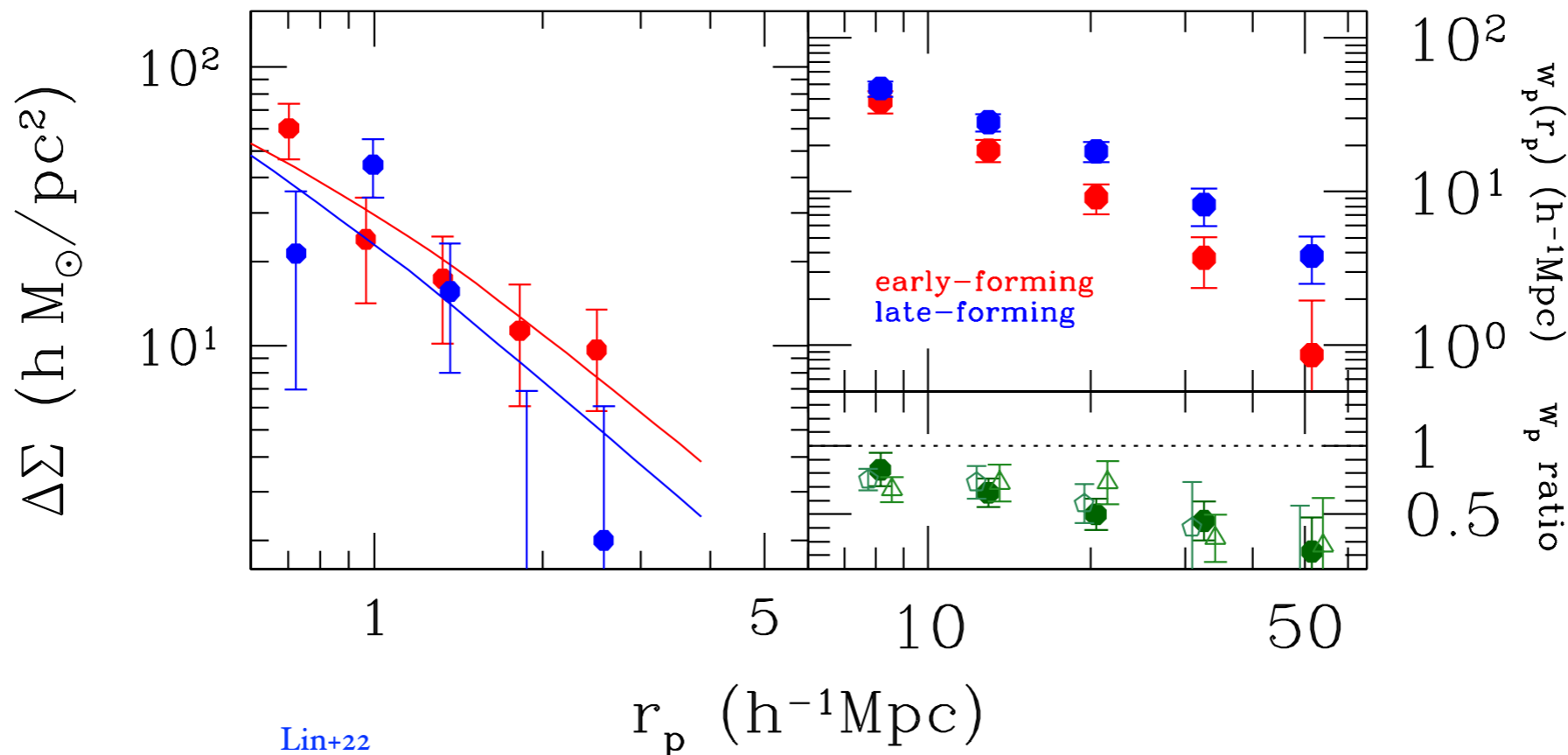
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- using the group catalog of Yang et al. (2007), Wang et al. (2016) have run a constrained simulation of the local Universe (SDSS DR7,  $z < 0.12$ ) called *Elucid*
- for structures larger than  $\sim 2 h^{-1} \text{Mpc}$ , there is very good correspondence between SDSS large scale structures and *Elucid* structures
- we have selected top  $\sim 630$  most massive clusters at  $z < 0.12$  from Yang's catalog
- MAH for each cluster is given by the counterpart halo in *Elucid*



# result: $z_{20}$

- clusters split by extrema in  $z_{20}$  *and* limited in mass and redshift: consider oldest and youngest clusters (**138 oldest =  $z_{20} > 1.35$**  ; **121 youngest =  $z_{20} < 0.85$** ) with  $\log M_{200m} = 14-14.5$  and  $z = 0.06-0.12$
- lensing masses *consistent within  $1\sigma$* :  $M_{200m,e} = (1.3 \pm 0.3) \times 10^{14} h^{-1} M_{\odot}$ ;  $M_{200m,l} = (1.0 \pm 0.3) \times 10^{14} h^{-1} M_{\odot}$
- large-scale clustering differs significantly ( $p = 1 \times 10^{-10}$ ) even after accounting for differences in mass: *we have a strong detection!!!*

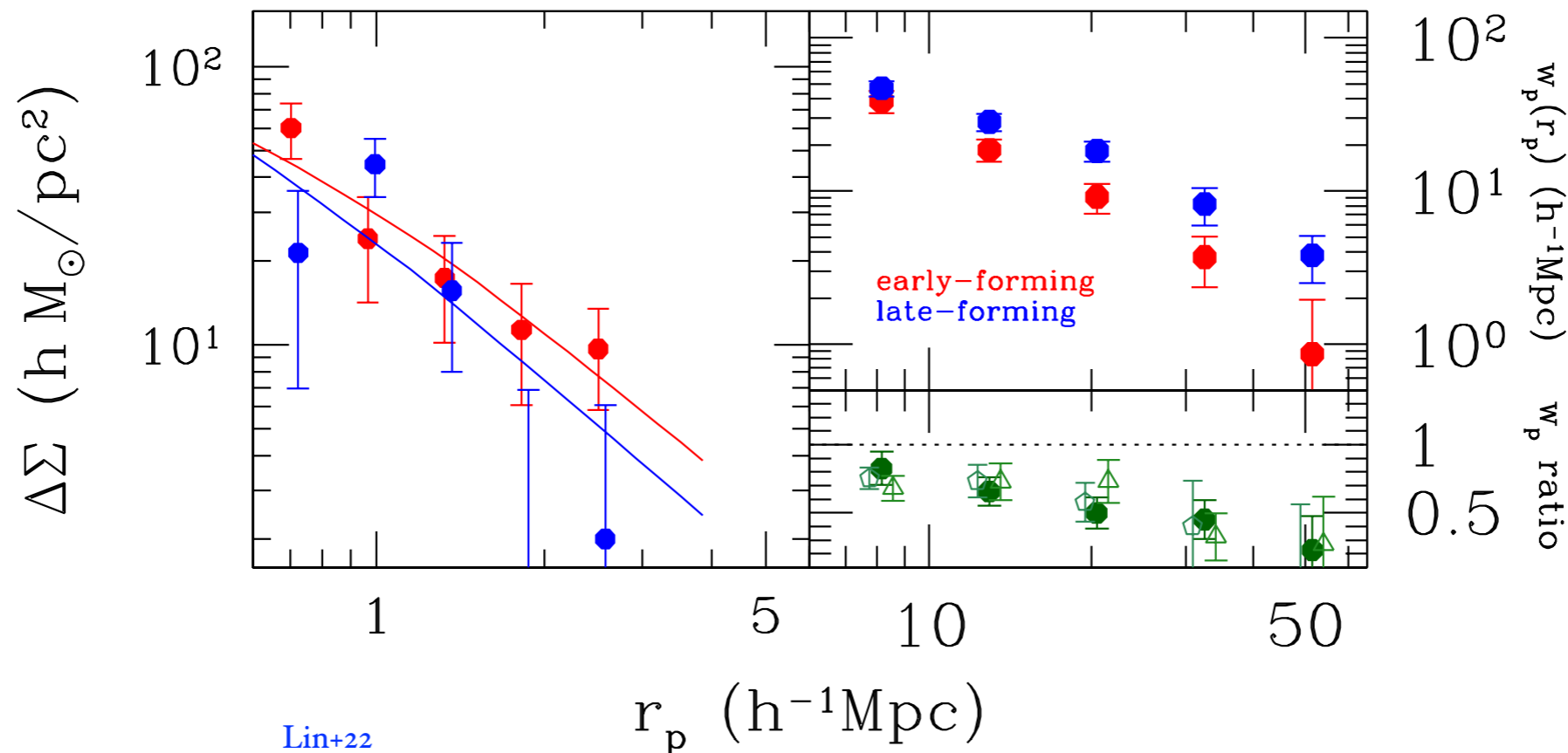




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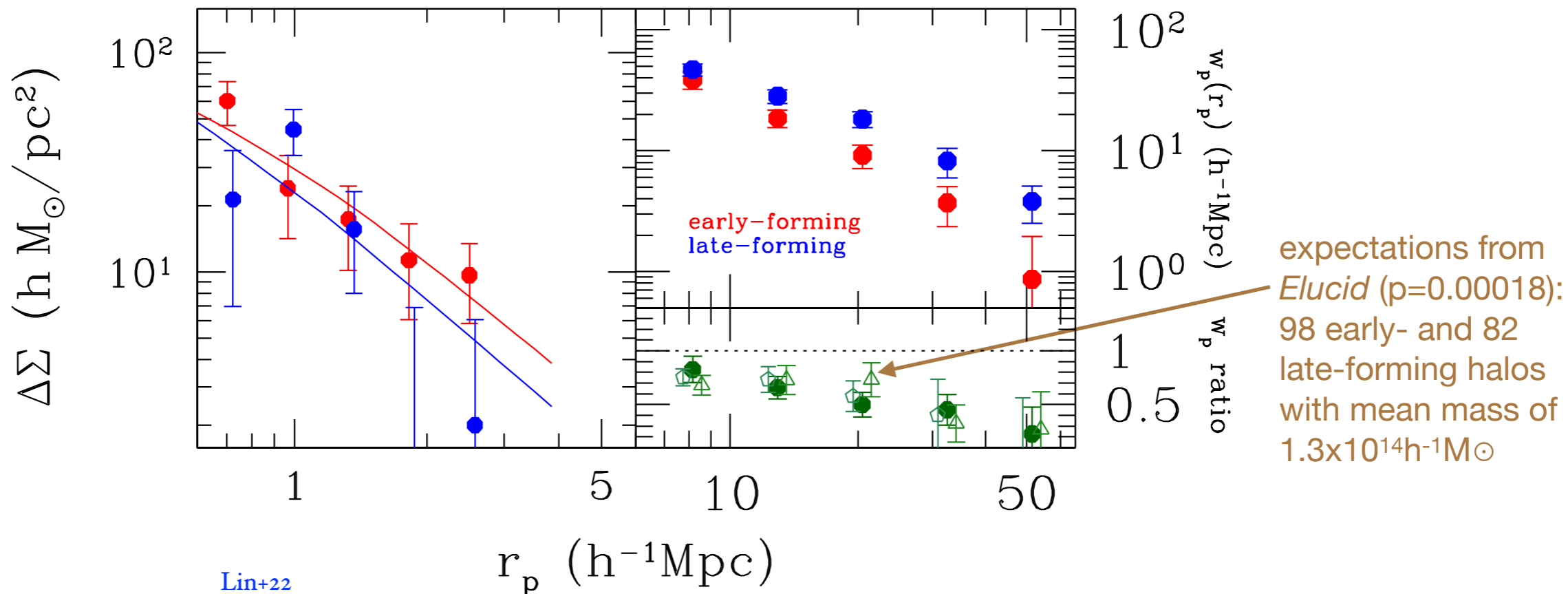




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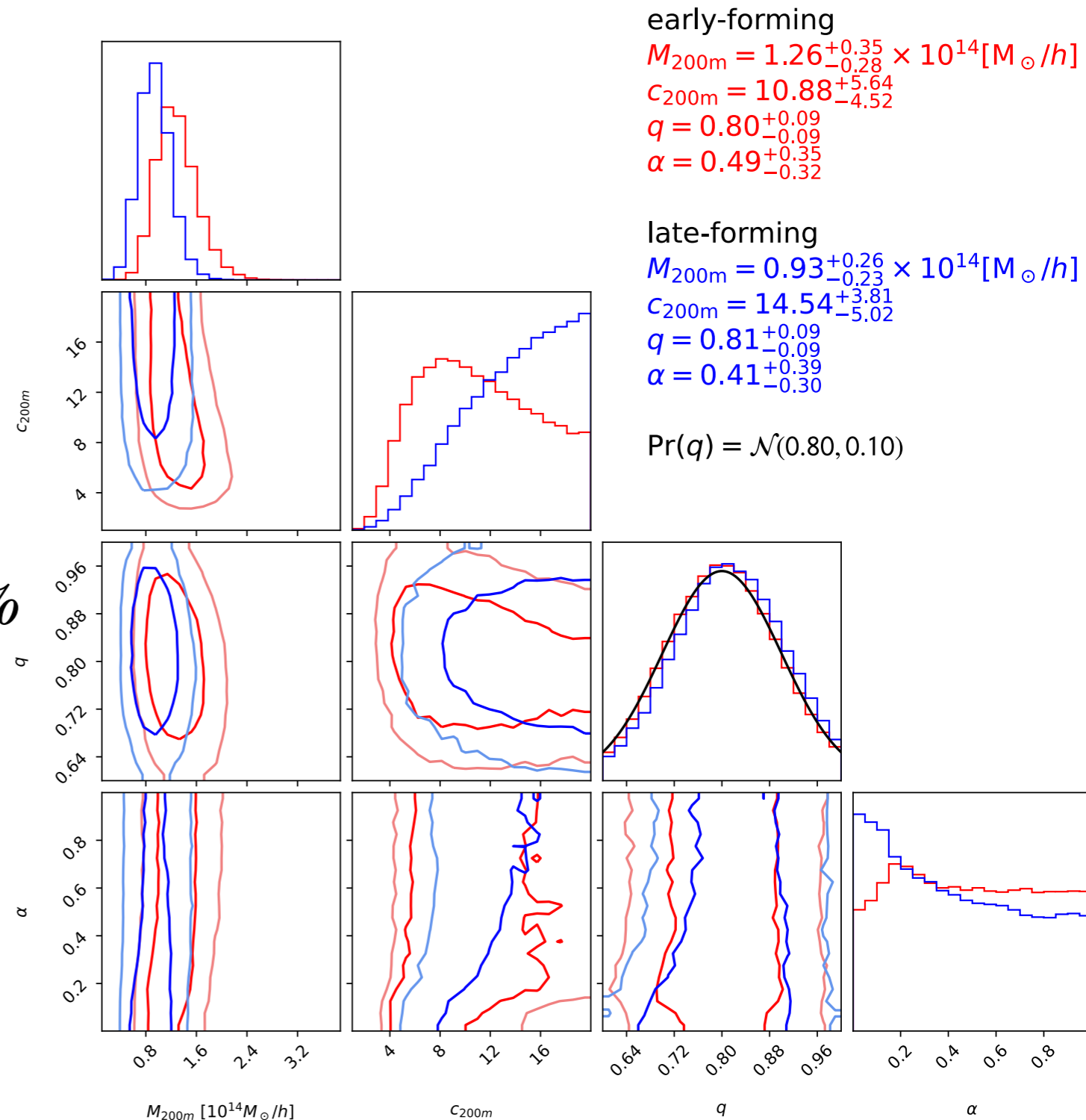
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# robustness of WL measurements?

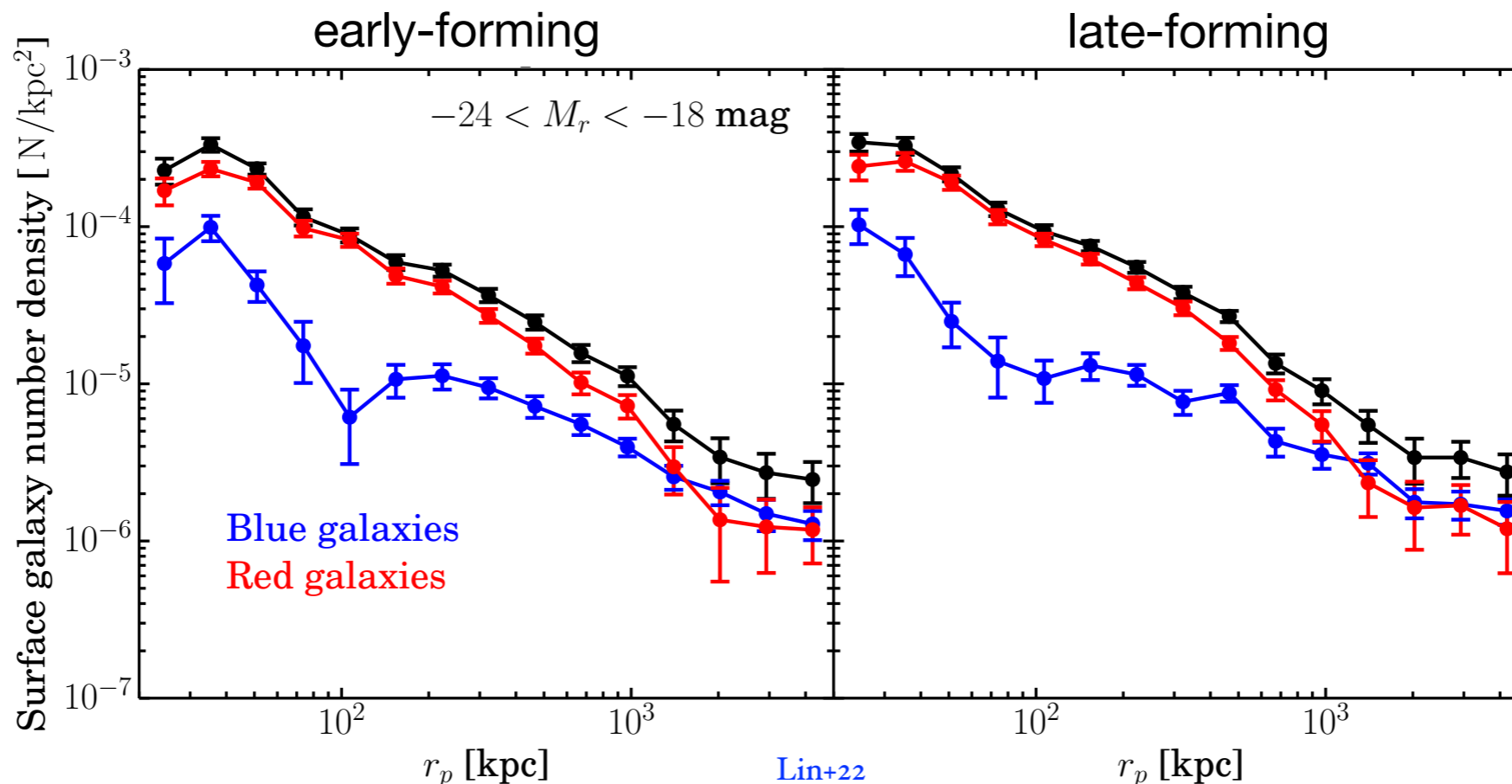
- what if our WL mass measurements were off?
  - maybe the early-forming sample mass is biased high by  $1\sigma$ , while that of late-forming sample is biased low by  $1\sigma$  (2.6% chance)  $\Rightarrow p=7.5 \times 10^{-5}$
  - *or* the late-forming sample mass is biased low by  $\geq 2\sigma$  (2.3% chance)  $\Rightarrow p=2.4 \times 10^{-5}$
  - if we assume 10% uncertainty in the Tinker+08 bias-halo mass relation and artificially decrease the expected biases, the probabilities become 0.0053 and 0.0025 (about  $3\sigma$  events)





# properties of galaxy populations

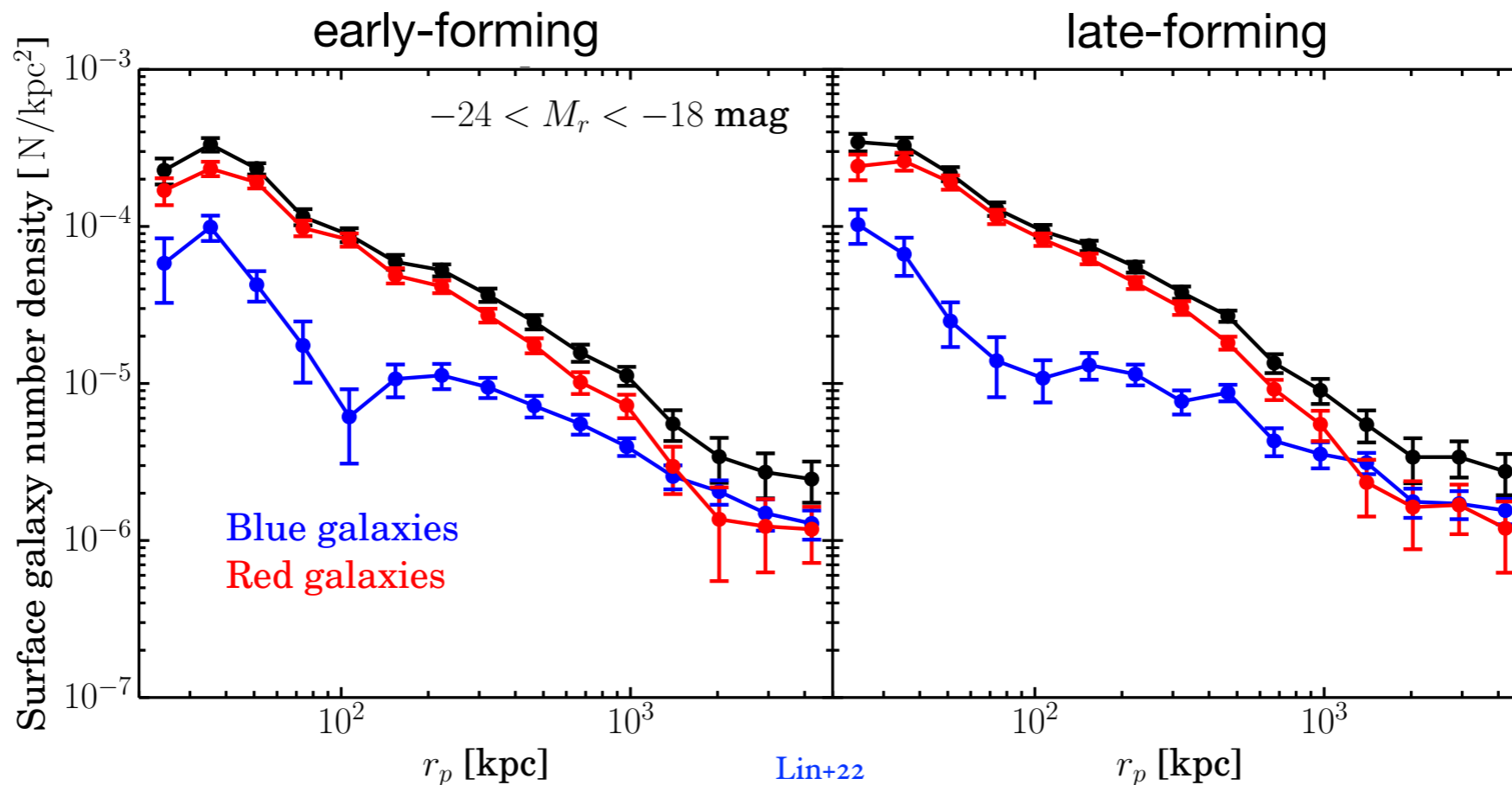
- using a cluster-galaxy cross-correlation technique, we derive surface density profiles of member galaxies of the two samples
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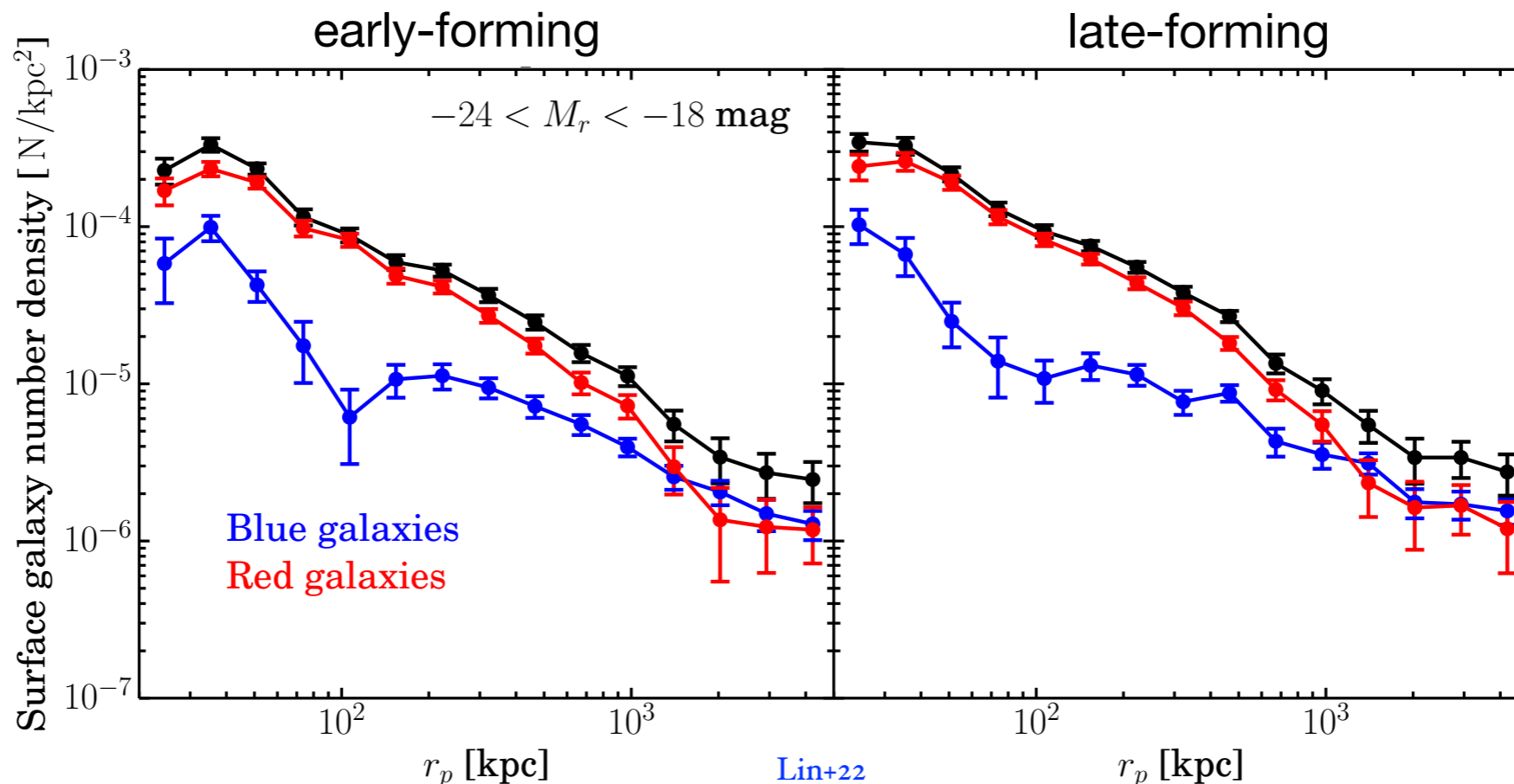
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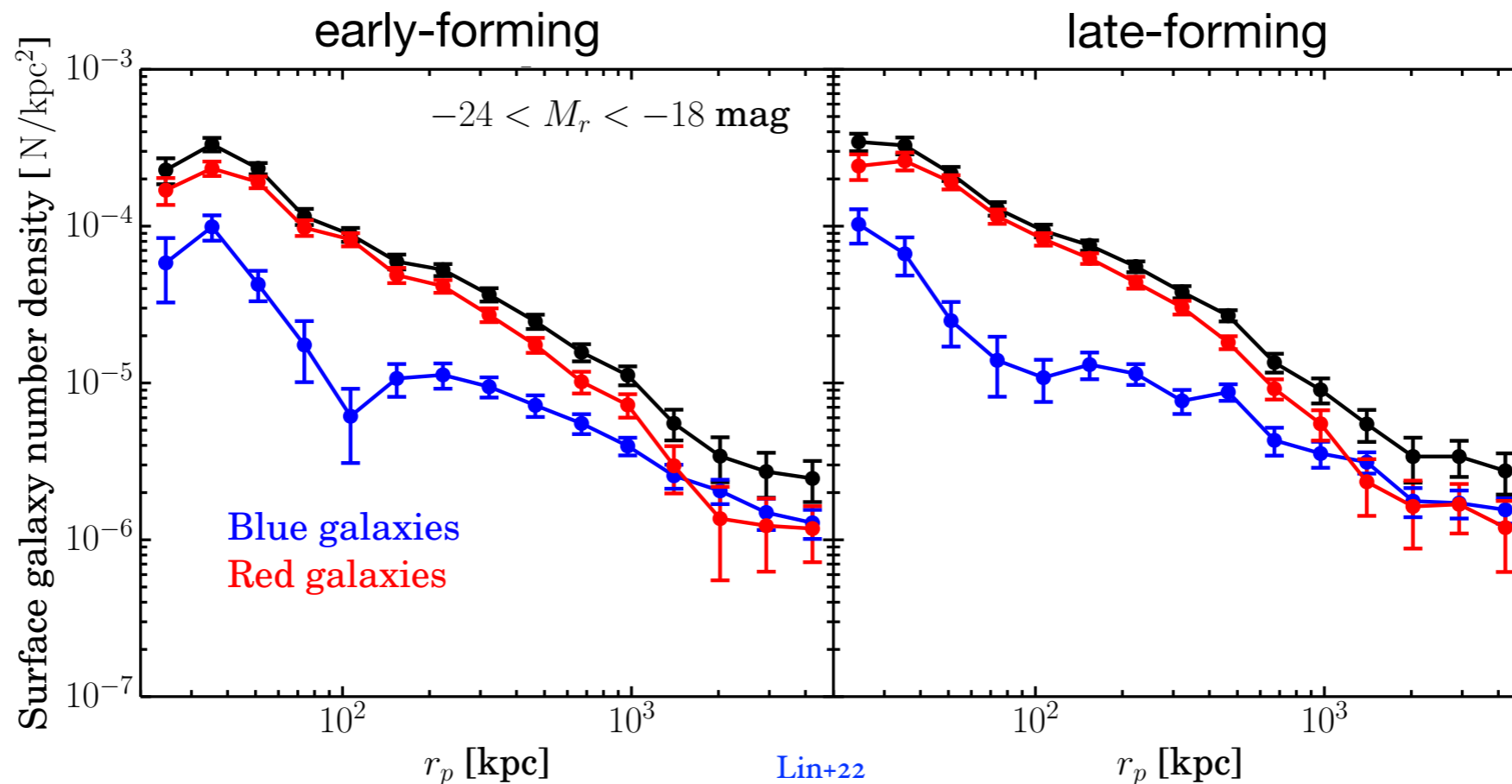
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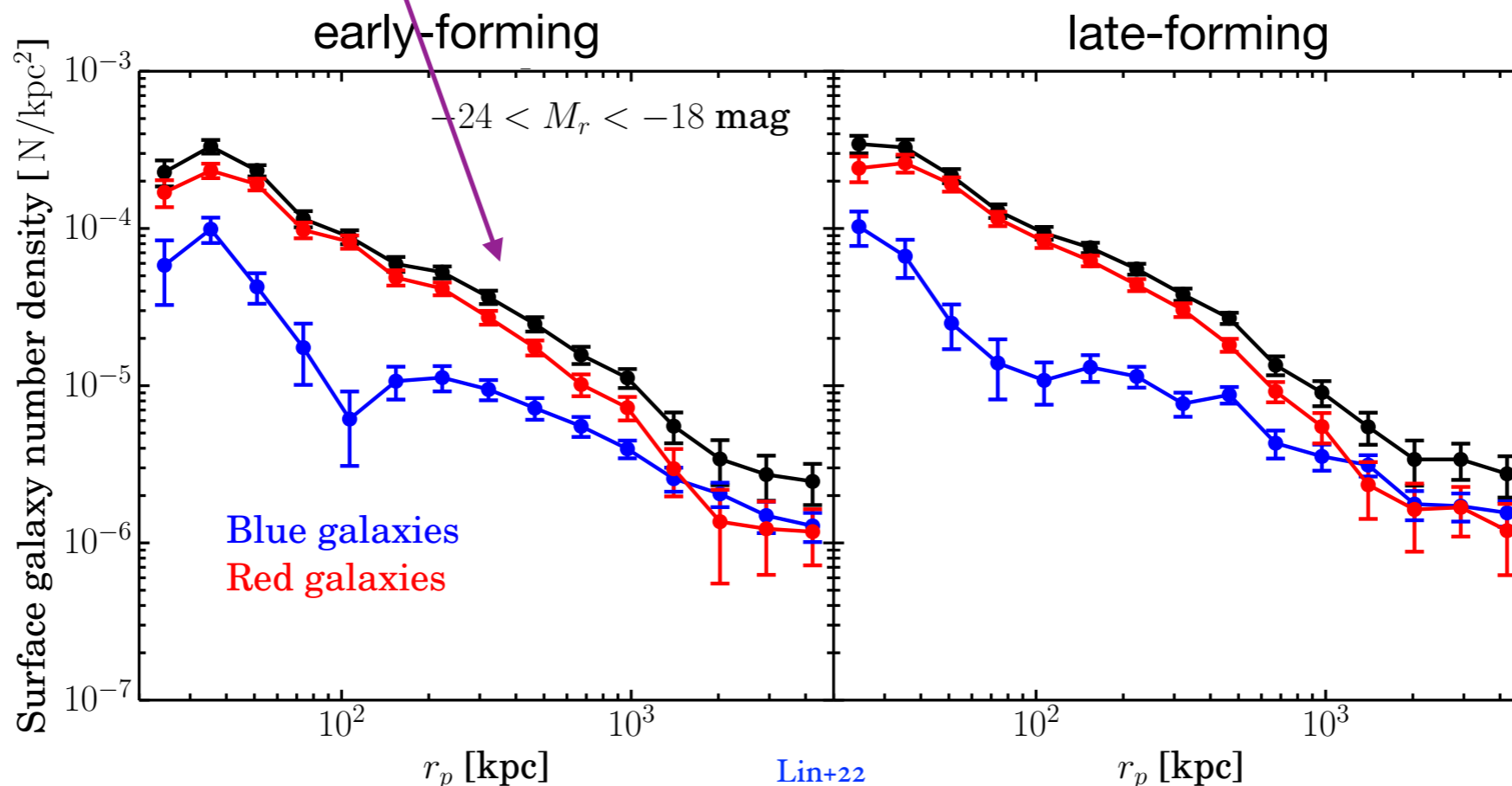
60% fewer than late-forming sample (considering cluster mass difference) if there is no AB

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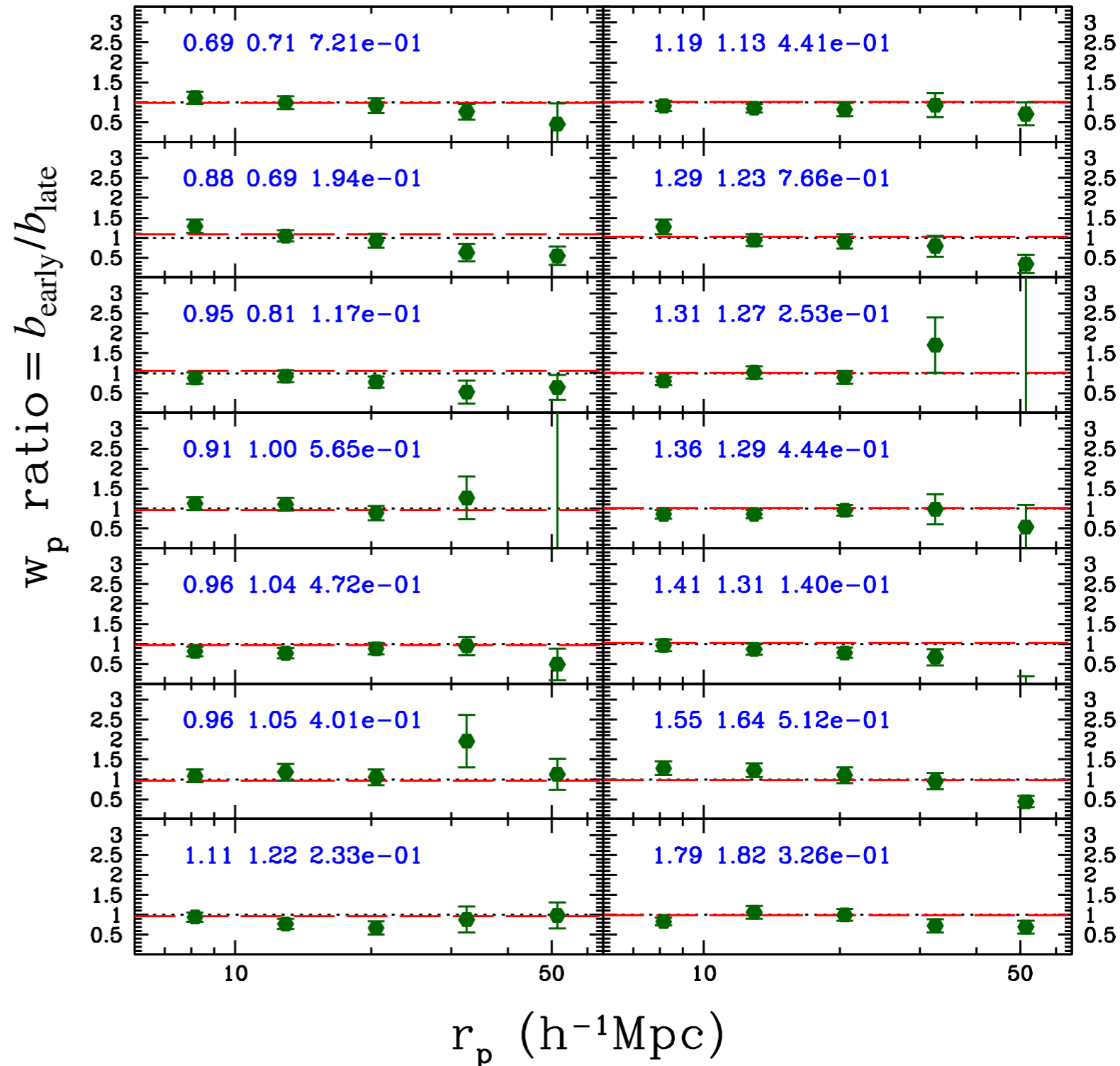


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  - concentrations of *red* galaxy distribution for the early- and late-forming clusters are  $c_e=7.1\pm 1.7$  and  $c_l=5.6\pm 0.6$
- no appreciable differences in mean age of brightest cluster galaxy (BCG) and other member galaxies are detected using full spectral or spectral energy distribution fitting methods
- median magnitude gap between BCG and G<sub>2</sub> (2nd most luminous galaxy):  $\Delta_e=0.44\pm 0.01$ ,  $\Delta_l=0.38\pm 0.01$ 
  - between BCG and G<sub>4</sub>:  $\Delta_e=0.99\pm 0.01$ ,  $\Delta_l=0.87\pm 0.01$
- median offset of BCG from cluster center:  $d_e=(0.11\pm 0.01)r_{200m}$ ,  $d_l=(0.14\pm 0.01)r_{200m}$
- *all* of these are consistent with the notion that the early-forming sample is indeed older, allowing BCGs to settle to the center and accrete more masses via galactic cannibalism

# null tests

- we have constructed 14 pairs of random cluster samples that have similar distribution in mass & z
- 3 numbers in blue: masses of early- & late-analog clusters,  $p$  for  $w_p$  to be consistent with theory (red line)
- none shows signals as strong as our samples
- mean BCG offset:  
 $d_e = (0.20 \pm 0.01)r_{200m}$ ,  
 $d_l = (0.22 \pm 0.01)r_{200m}$
- median magnitude gap:  
 $\Delta_e = 0.42 \pm 0.01$ ,  $\Delta_l = 0.42 \pm 0.01$





# a Bayesian way of thinking about this

- are we measuring AB in the real world or only in *Elucid*?
- recast our study as a hypothesis test  $\Rightarrow$  ruling out the null hypothesis that *AB does not exist in the Universe*
- $P(\text{AB} \mid \text{data}, \text{Elucid}) \propto P(\text{data} \mid \text{AB}, \text{Elucid})P(\text{AB} \mid \text{Elucid})$ 
  - $\text{AB} \equiv$  “AB exists in the Universe”
  - data  $\equiv$  properties of our cluster samples (WL, clustering, cluster galaxy properties)
  - prior  $P(\text{AB} \mid \text{Elucid}) = 1$  or  $0$  (each 50%)

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	AB exists in real Universe	
AB exists in Elucid	yes $\otimes$ yes	yes $\otimes$ no
	no $\otimes$ yes	no $\otimes$ no

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	↓	↓
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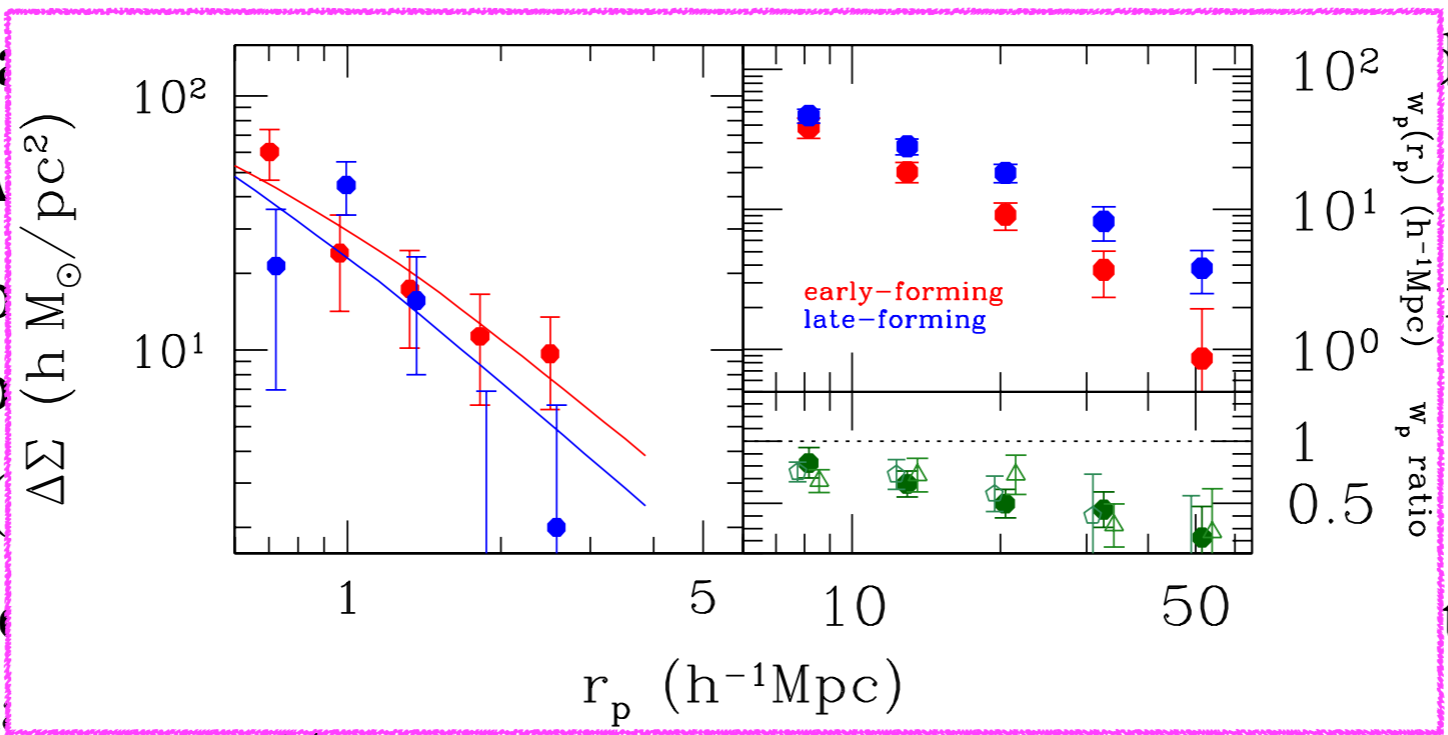
- $P(\text{AB} \mid \text{data})$

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- $\text{data} \equiv \text{p galaxy p}$

- prior  $P(\Delta\Sigma)$

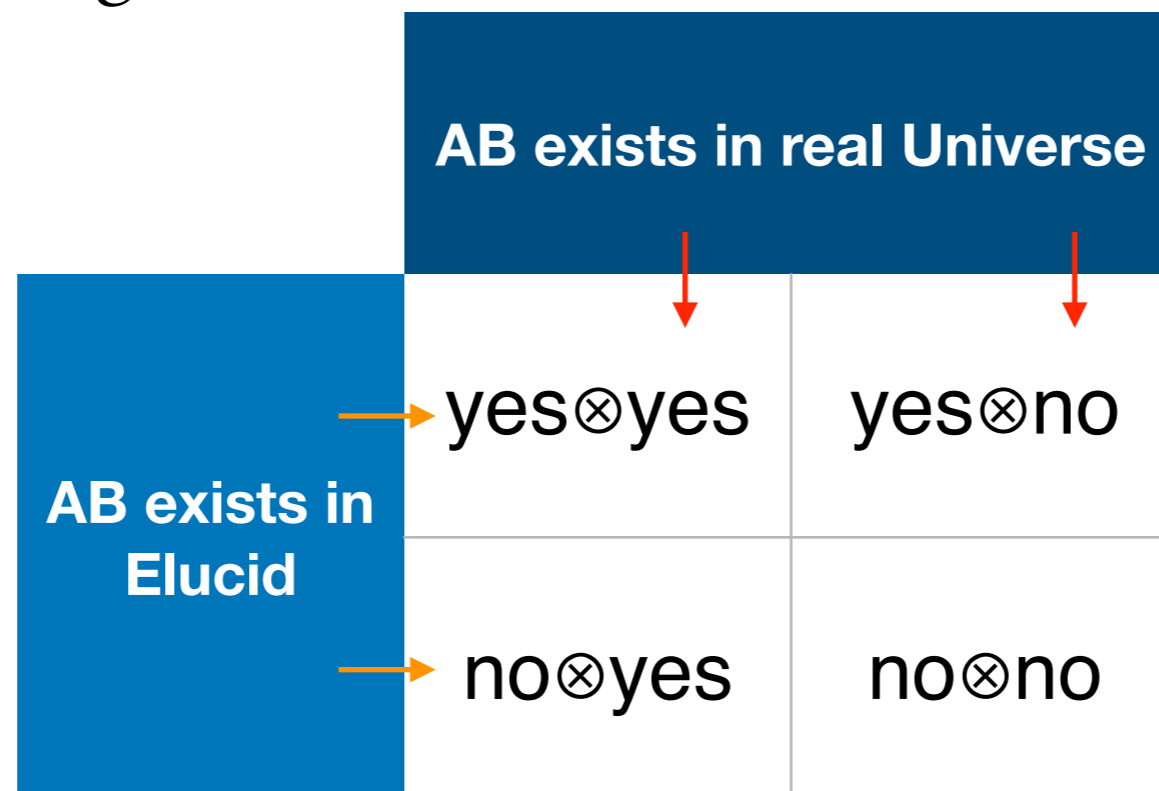
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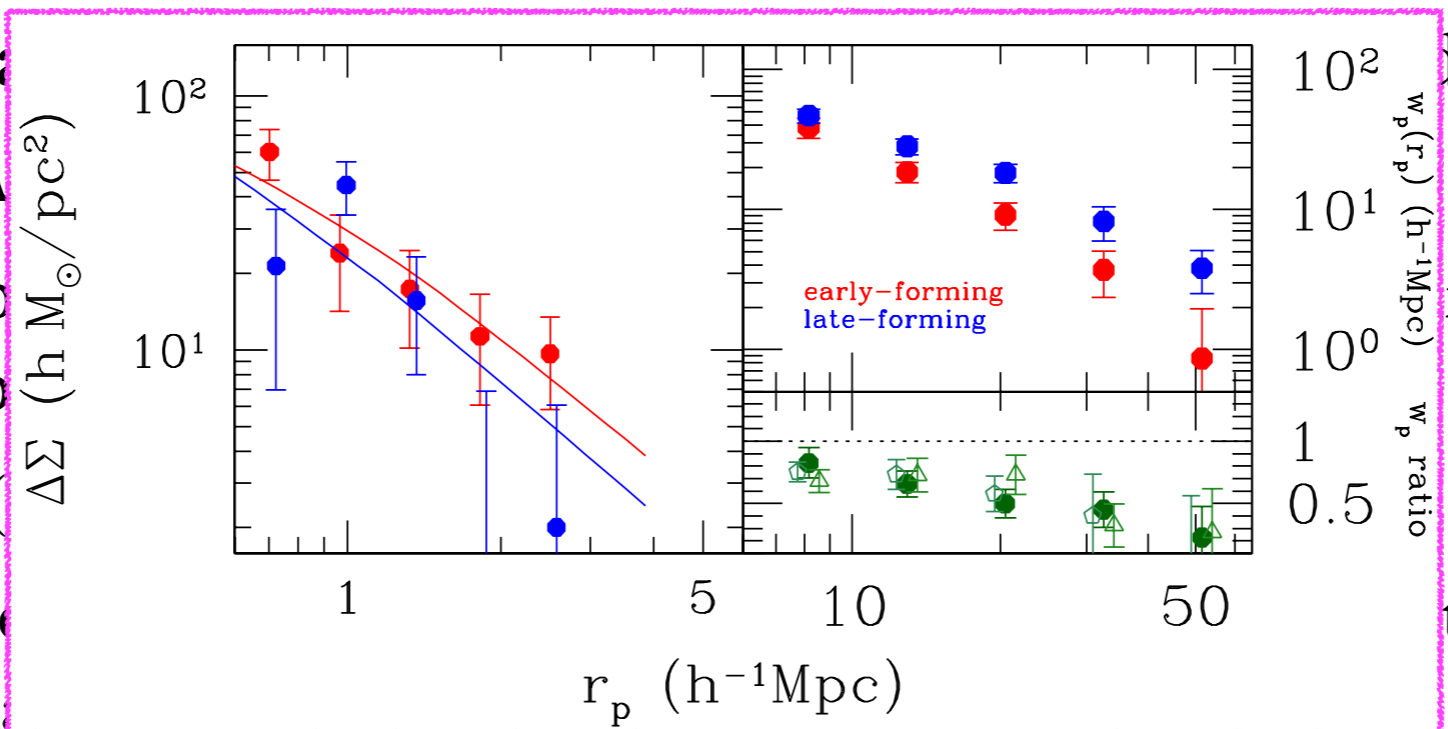
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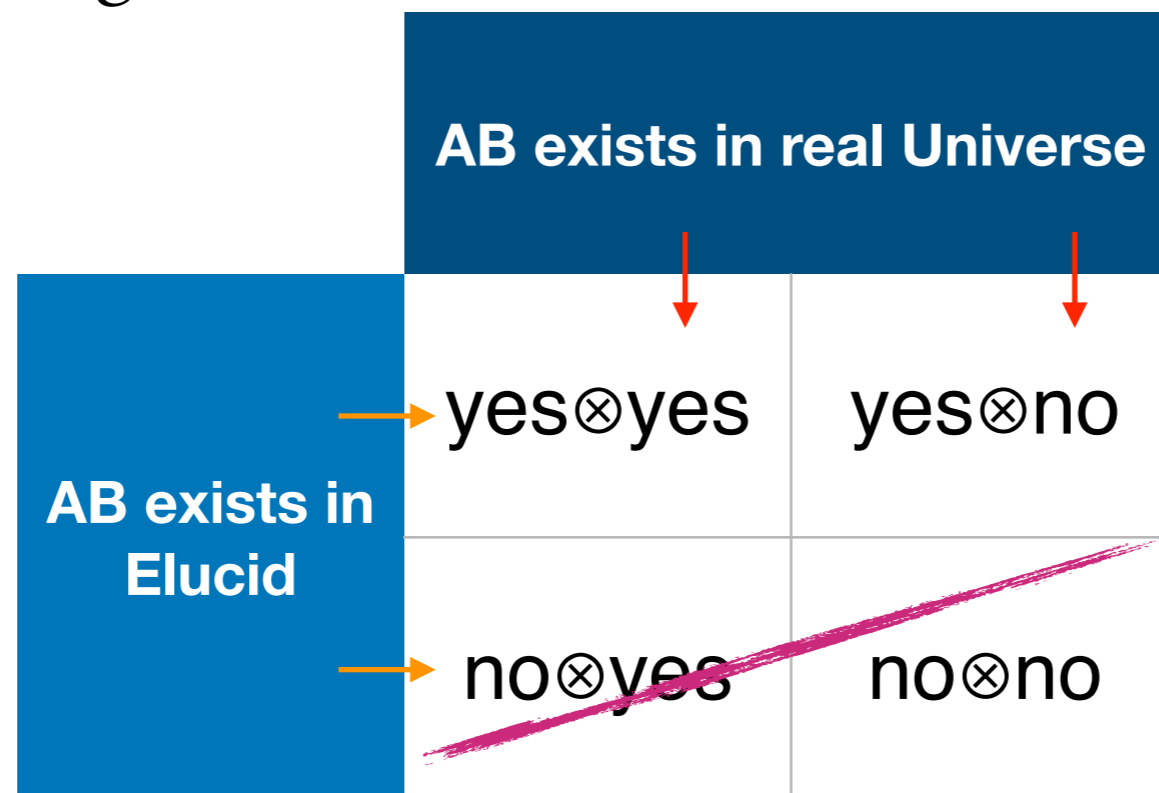
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Observable	True AB	Observed trend	Spurious AB due to circularity	Spurious AB due to incorrect cluster mass
Concentration	$c_e > c_l$	$c_e > c_l$	$c_e = c_l$	$c_e \approx c_l$
Galaxy number	$N_e < N_l$	$N_e < N_l$	$N_e = N_l$	$N_e \lesssim N_l$
BCG offset	$d_e < d_l$	$d_e < d_l$	$d_e = d_l$	$d_e > d_l$
Magnitude gap	$\Delta_e > \Delta_l$	$\Delta_e > \Delta_l$	$\Delta_e = \Delta_l$	$\Delta_e > \Delta_l$

- circularity: *Elucid* is built from the density field based on group and cluster catalog of Yang+07, so *Elucid* halos are expected to be in LSS similar to Yang+07 clusters  $\Rightarrow z_{20}$  only meaningful in *Elucid*

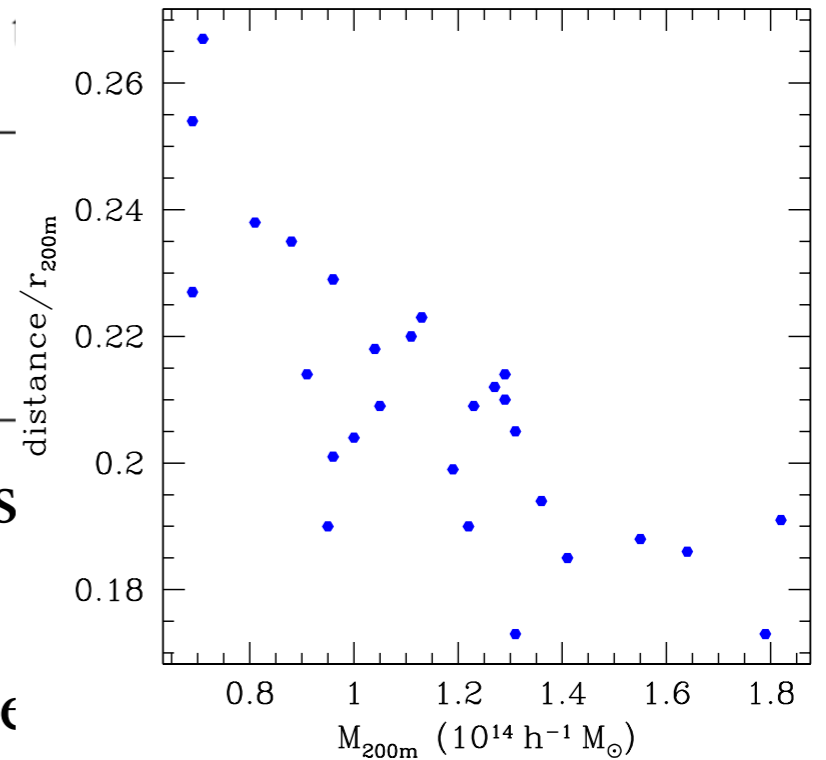
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- if cluster mass of our late-forming sample is severely underestimated
  - concentration varies very weakly with cluster mass  $\Rightarrow c_e \approx c_1$
  - $N \propto M^{0.8}$  (but with large scatter)  $\Rightarrow N_e \lesssim N_1$
  - BCG offset decreases with increasing cluster mass  $\Rightarrow d_e > d_1$
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  - concentration varies very weakly with cluster mass  $\Rightarrow c_e \approx c_1$
  - $N \propto M^{0.8}$  (but with large scatter)  $\Rightarrow N_e \lesssim N_1$
  - BCG offset decreases with increasing cluster mass  $\Rightarrow d_e > d_1$
  - mag gap is found to decrease with cluster mass (Lin+IO)  $\Rightarrow \Delta_e > \Delta_1$



# a Bayesian way of thinking about this

Observable	True AB	Observed trend	Spurious AB due to circularity	Spurious AB due to incorrect cluster mass
Concentration	$c_e > c_1$	$c_e > c_1$	$c_e = c_1$	$c_e \approx c_1$
Galaxy number	$N_e < N_1$	$N_e < N_1$	$N_e = N_1$	$N_e \lesssim N_1$
BCG offset	$d_e < d_1$	$d_e < d_1$	$d_e = d_1$	$d_e > d_1$
Magnitude gap	$\Delta_e > \Delta_1$	$\Delta_e > \Delta_1$	$\Delta_e = \Delta_1$	$\Delta_e > \Delta_1$

- if cluster mass of our late-forming sample is severely underestimated
  - concentration varies very weakly with cluster mass  $\Rightarrow c_e \approx c_1$
  - $N \propto M^{0.8}$  (but with large scatter)  $\Rightarrow N_e \lesssim N_1$
  - BCG offset decreases with increasing cluster mass  $\Rightarrow d_e > d_1$
  - mag gap is found to decrease with cluster mass (Lin+IO)  $\Rightarrow \Delta_e > \Delta_1$
- none of the 14 pairs of control/random samples passes these tests

# a Bayesian way of thinking about this

	<b>AB exists in real Universe</b>	
<b>AB exists in Elucid</b>	yes ⊗ yes	<del>yes ⊗ no</del>
	<del>no ⊗ yes</del>	no ⊗ no

Spurious AB due to circularity	Spurious AB due to incorrect cluster mass
$c_e = c_1$	$c_e \approx c_1$
$N_e = N_1$	$N_e \lesssim N_1$
$d_e = d_1$	$d_e > d_1$
$\Delta_e = \Delta_1$	$\Delta_e > \Delta_1$

ing sample is severely

ly with cluster mass  $\Rightarrow c_e \approx c_1$

- $N \propto M^{0.0}$  (but with large scatter)  $\Rightarrow N_e \lesssim N_1$
- BCG offset decreases with increasing cluster mass  $\Rightarrow d_e > d_1$
- mag gap is found to decrease with cluster mass (Lin+IO)  $\Rightarrow \Delta_e > \Delta_1$
- none of the 14 pairs of control/random samples passes these tests
- we live in the “yes” ⊗ “yes” box!

# a Bayesian way of thinking about this

	AB exists in real Universe		Spurious AB due to circularity	Spurious AB due to incorrect cluster mass
AB exists in Elucid	yes ⊗ yes	<del>yes ⊗ no</del>	$c_e = c_1$ $N_e = N_1$ $d_e = d_1$ $\Delta_e = \Delta_1$	$c_e \approx c_1$ $N_e \lesssim N_1$ $d_e > d_1$ $\Delta_e > \Delta_1$
	<del>no ⊗ yes</del>	<del>no ⊗ no</del>		

late-forming sample is severely

underestimated with cluster mass  $\Rightarrow c_e \approx c_1$

- $N \propto M^{0.0}$  (but with large scatter)  $\Rightarrow N_e \lesssim N_1$
- BCG offset decreases with increasing cluster mass  $\Rightarrow d_e > d_1$
- mag gap is found to decrease with cluster mass (Lin+10)  $\Rightarrow \Delta_e > \Delta_1$
- none of the 14 pairs of control/random samples passes these tests
- we live in the “yes” ⊗ “yes” box!
- *even if the mean mass of our late-forming sample is truly severely underestimated, the difference in masses is still far from sufficient to explain the huge difference in biases  $\Rightarrow$  something like AB at work*

# prospects

- among the first group to show a firm detection of assembly bias signal at cluster-scale halos: an important validation of  $\Lambda$ CDM
  - can study other aspects of assembly bias: spin or concentration
  - can further examine differences in intracluster medium
  - hard to detect splashback radius due to small sample size
- *it is still imperative to find ways that are more directly linked to observations to label clusters as early- or late-forming*
  - construct early- and late-forming samples using observable trends found in our study (member galaxy spatial concentration, galaxy number, BCG offset, magnitude gap...)
- forward-modeling techniques like that employed by *Elucid* are becoming popular (e.g., *BORG*, *TARDIS*, *COSMIC BIRTH*)
  - rich spectroscopic datasets from DESI and PFS will allow us to do reconstruction at high- $z$ : studying assembly bias/galactic conformity!

please see Lin, Miyatake et al. (2022, *A&A*, 666, A97) for more details!