上海交通大學 SHANGHAI JIAO TONG UNIVERSITY

SJTUPA 上海交通大学物理与天文学院

上海交通大学天文系 Astronomy at Shanghai Jiao Tong University

AKRA: Accurate Kappa Reconstruction Algorithm for masked shear catalog

Presenter: Shi Yuan (Postdoc at SJTU)

1-Nov-2023

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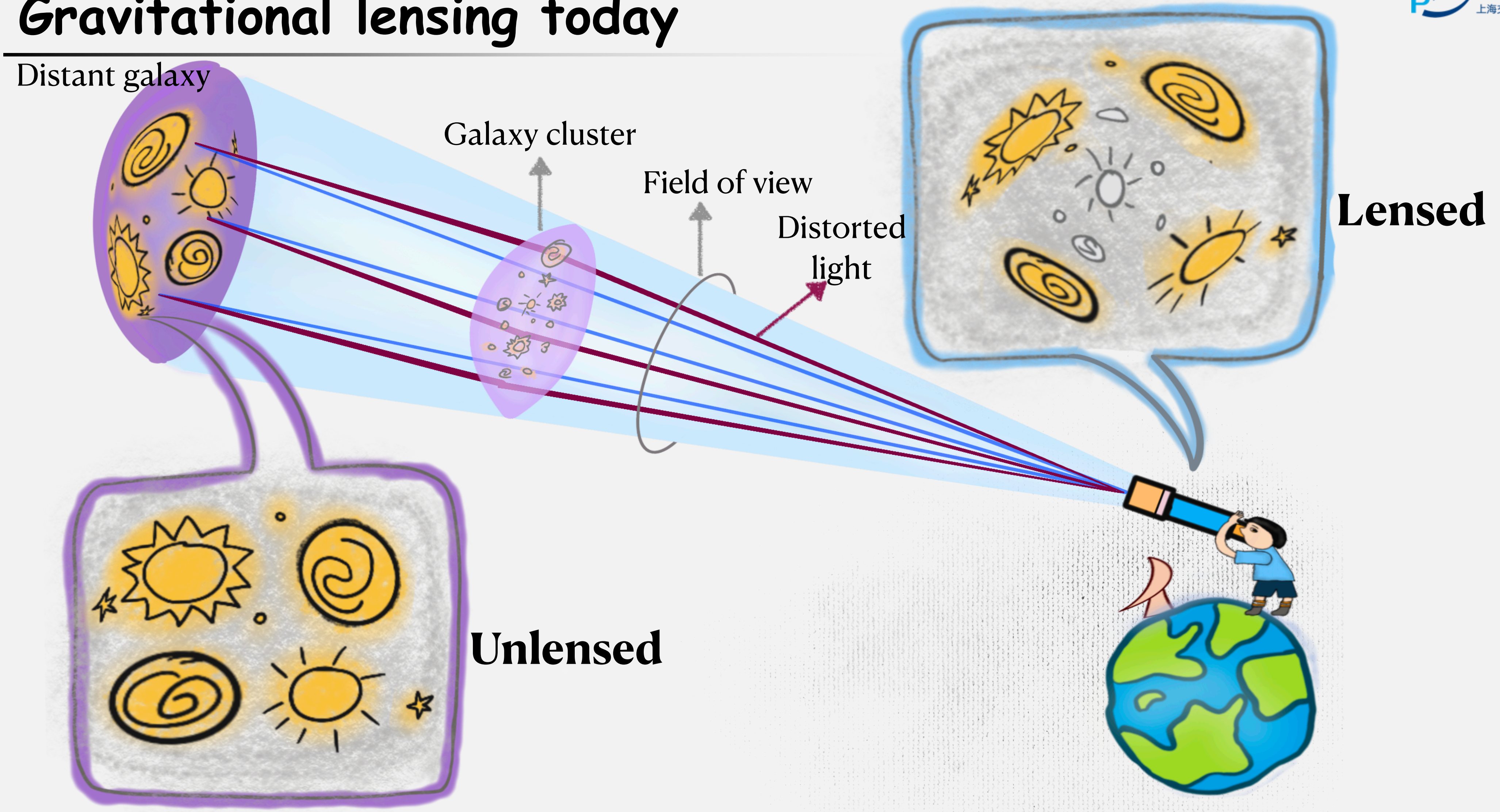
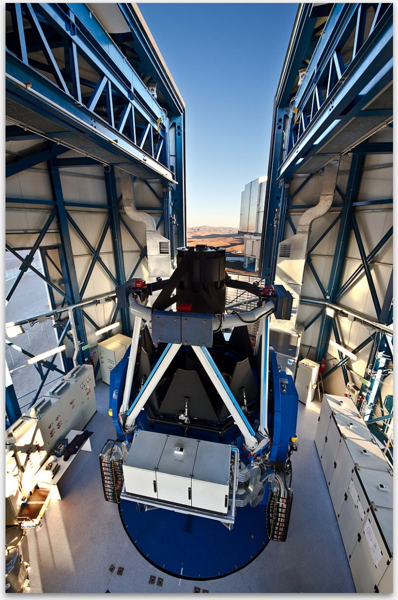


Steps in a cosmic journey



Stage III (Ongoing)

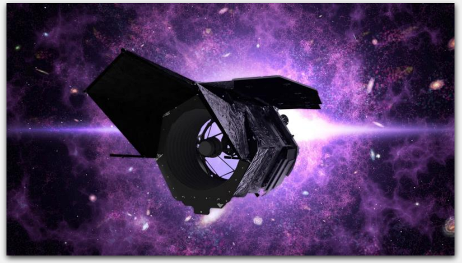
S/N $\gtrsim$ 30

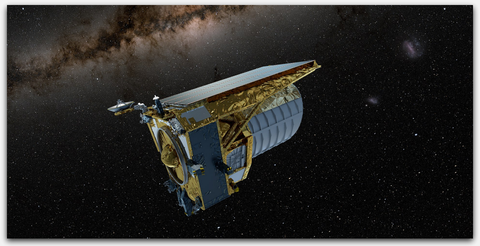
KiDS, DES, Subaru, HSC...

CSST, LSST,
Euclid,
Roman (WFIRST) ...

Stage IV (upcoming)

S/N $\gtrsim$ 400



Signal-to-noise ratio (S/N)
in cosmic shear measurement

Kaiser-Squire (KS) method

Real Space

Fourier Space

$$\kappa = \frac{1}{2}(\psi_{11} + \psi_{22}) \Rightarrow \tilde{\kappa} = -\frac{1}{2}(k_1^2 + k_2^2)\tilde{\psi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \Rightarrow \tilde{\gamma}_1 = -\frac{1}{2}(k_1^2 - k_2^2)\tilde{\psi}$$

$$\gamma_2 = \psi_{12} \Rightarrow \tilde{\gamma}_2 = -k_1 k_2 \tilde{\psi}$$

Observables

$$\begin{bmatrix} \gamma_1(\vec{\ell}) \\ \gamma_2(\vec{\ell}) \end{bmatrix} = \begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix} \cdot \tilde{\kappa}(\vec{\ell})$$

Using: $\begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix} \cdot \begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix}^T = 1$

$$\kappa(\vec{\theta}) = \frac{1}{\pi}\int \mathrm{d}^2\theta' \left[D_1\left(\vec{\theta} - \vec{\theta}'\right)\gamma_1 + D_2\left(\vec{\theta} - \vec{\theta}'\right)\gamma_2\right]$$

$$\widetilde{(f * g)} = \tilde{f}\tilde{g}$$

$$\tilde{\kappa}(\vec{\ell}) = \begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix}^T \cdot \begin{pmatrix} \gamma_1(\vec{\ell}) \\ \gamma_2(\vec{\ell}) \end{pmatrix} \rightarrow \tilde{E}(\ell)$$

$$\tilde{B}(\vec{\ell}) = \tilde{\gamma}_1(\vec{\ell})\sin(2\phi_1) - \tilde{\gamma}_2(\vec{\ell})\cos(2\phi_\ell) = 0$$

Kaiser-Squire (KS) method

$$\Rightarrow \quad \tilde{\kappa} = -\frac{1}{2}\left(k_1^2 + k_2^2\right)\tilde{\psi}$$

$$\Rightarrow \quad \tilde{\gamma}_1 = -\frac{1}{2}\left(k_1^2 - k_2^2\right)\tilde{\psi}$$

$$\Rightarrow \quad \tilde{\gamma}_2 = -k_1 k_2 \tilde{\psi}$$

$$\begin{bmatrix} \gamma_1(\vec{\ell}) \\ \gamma_2(\vec{\ell}) \end{bmatrix} = \begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix} \cdot \tilde{\kappa}(\vec{\ell})$$

Using: $\begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix} \cdot \begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix}^T = 1$

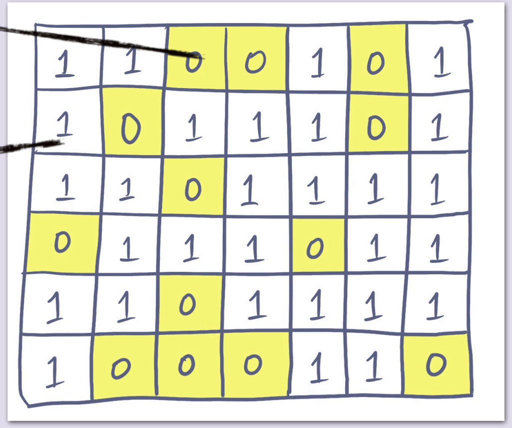
$$\tilde{\kappa}(\vec{\ell}) = \begin{bmatrix} \cos(2\phi_{\ell_1}) \\ \sin(2\phi_{\ell_1}) \end{bmatrix}^T \cdot \begin{pmatrix} \gamma_1(\vec{\ell}) \\ \gamma_2(\vec{\ell}) \end{pmatrix} \rightarrow \tilde{E}(\ell)$$

$$\tilde{B}(\vec{\ell}) = \tilde{\gamma}_1(\vec{\ell})\sin(2\phi_1) - \tilde{\gamma}_2(\vec{\ell})\cos(2\phi_\ell) = 0$$

If mask exists ...

masked pixels

unmask pixels

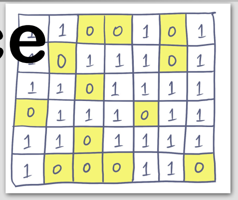


$$\gamma_i^M(\vec{\theta}) = m(\vec{\theta})\gamma_i(\vec{\theta})$$

$$\tilde{B}(\vec{\ell}) = \tilde{\gamma}_1^M(\vec{\ell})\sin(2\phi_1) - \tilde{\gamma}_2^M(\vec{\ell})\cos(2\phi_\ell) \neq 0$$

AKRA

Real Space



Mask function $m(\theta)$

$$\gamma_i^M(\vec{\theta}) = m(\vec{\theta})\gamma_i(\vec{\theta}) \quad \Rightarrow$$

Fourier Space

$$\tilde{\gamma}_i^m(\vec{L}) = \int \gamma_1(\vec{\theta})m(\vec{\theta})e^{-i\vec{L}\cdot\vec{\theta}}d^2\theta$$

$$= \int \frac{d^2\ell_1}{(2\pi)^2}\int d^2\ell_2\tilde{\gamma}_i\left(\vec{\ell}_1\right)\tilde{m}\left(\vec{\ell}_2\right)\delta^D\left(\vec{\ell}_1+\vec{\ell}_2-\vec{L}\right)$$

$$(\tilde{fg}) = \tilde{f} * \tilde{g}$$

$$\propto \int d^2\vec{\ell}_1\tilde{\gamma}_i\left(\vec{\ell}_1\right)\tilde{m}\left(\vec{L}-\vec{\ell}_1\right)$$

$$\tilde{\gamma}_i^m(\vec{L}) = \sum_{\vec{\ell}_1=1}^{N_\ell^2}\tilde{\gamma}_i\left(\vec{\ell}_1\right)\tilde{m}\left(\vec{L}-\vec{\ell}_1\right)\Delta\Omega$$

$$= \sum_{\vec{\ell}_1=1}^{N_\ell^2}\tilde{\gamma}_i\left(\vec{\ell}_1\right)M\left(\vec{\ell}_1\right)\Delta\Omega$$

$$\tilde{\gamma}_1^m(\vec{L}) = \mathbf{M}\cdot\tilde{\gamma}_1\left(\vec{\ell}_1\right), \quad \tilde{\gamma}_2^m(\vec{L}) = \mathbf{M}\cdot\tilde{\gamma}_2\left(\vec{\ell}_1\right)$$

N_ℓ^2 (N_ℓ^2, N_ℓ^2) N_ℓ^2

AKRA

$$\gamma_i^M(\vec{\theta}) = m(\vec{\theta})\gamma_i(\vec{\theta})$$

$$\tilde{\gamma}_1^m(\vec{L}) = \mathbf{M}\cdot\tilde{\gamma}_1\left(\vec{\ell}_1\right), \quad \tilde{\gamma}_2^m(\vec{L}) = \mathbf{M}\cdot\tilde{\gamma}_2\left(\vec{\ell}_1\right)$$

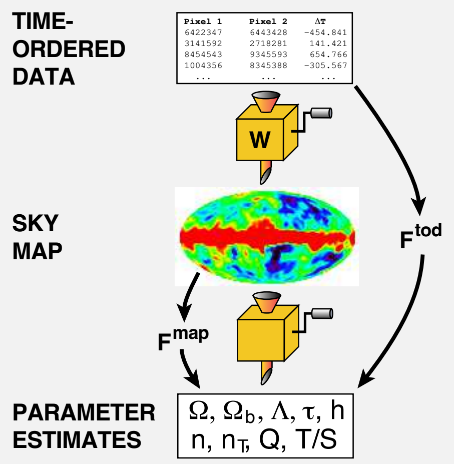
Using: $\begin{bmatrix}\gamma_1(\vec{\ell})\\ \gamma_2(\vec{\ell})\end{bmatrix} = \begin{bmatrix}\cos(2\phi_{\ell_1})\\ \sin(2\phi_{\ell_1})\end{bmatrix}\cdot\tilde{\kappa}(\vec{\ell})$

$$\begin{bmatrix}\tilde{\gamma}_1^m(\vec{L})\\ \tilde{\gamma}_2^m(\vec{L})\end{bmatrix} = \begin{bmatrix}\cos(2\phi_{\ell_1})\,\mathbf{M}\\ \sin(2\phi_{\ell_1})\,\mathbf{M}\end{bmatrix}\cdot\tilde{\kappa}\left(\vec{\ell}_1\right)$$

$2N_\ell^2$ $\quad 2N_\ell^2\times N_\ell^2$ $\quad 2N_\ell^2$

$$\boldsymbol{\gamma} = \mathbf{A}\boldsymbol{\kappa} + \boldsymbol{n}$$

Mapmaking in CMB experiments



$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}.$$
$$\tilde{\mathbf{x}} = \mathbf{W}\mathbf{y}$$

(Max Tegmark 1997)

Minimum variance estimate:

$$\hat{\boldsymbol{\kappa}} = \mathbf{D}\mathbf{A}^T\mathbf{N}^{-1}\boldsymbol{\gamma}.$$

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$$\gamma = \mathbf{A}\kappa + n$$

The minimum variance estimator:

$$\hat{\kappa} = \mathbf{D}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma.$$

The ensemble average of the estimator:

$$\langle\hat{\kappa}\rangle = \langle \mathbf{D}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma\rangle = \mathbf{D}\left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)\kappa \equiv \mathbf{P}\kappa,$$

→ Define PSF

The covariance of the estimator:

$$\mathbf{C} \equiv \left\langle(\hat{\kappa}-\kappa)(\hat{\kappa}-\kappa)^T\right\rangle = \mathbf{P}\mathbf{D}^{\mathrm{T}}$$

Ideal case: $\mathbf{D} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}$

$$\mathbf{C} = \mathbf{P}\mathbf{D}^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}$$

$$\hat{\kappa}^{\mathbf{R}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A} + \mathbf{R}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma$$

$\mathbf{R} = \varepsilon\mathbf{I}$ **ensures numerical stability**

Assumption:
1. flat sky,
2. noise-free,
3. periodic boundary.

1. Generation of convergence field

2. Generation of shear field

3. Adding a mask $m(\theta)$ to the shear field $\gamma_1^M(\theta), \gamma_2^M(\theta)$

4. Generation of convolution kernel matrix $\mathbf{M} = M(\vec{\ell}_1)$

5. Modification of convolution kernel matrix:

$$\mathbf{A} = \begin{bmatrix}\cos(2\phi_{\ell_1})\,\mathbf{M}\\ \sin(2\phi_{\ell_1})\,\mathbf{M}\end{bmatrix}$$

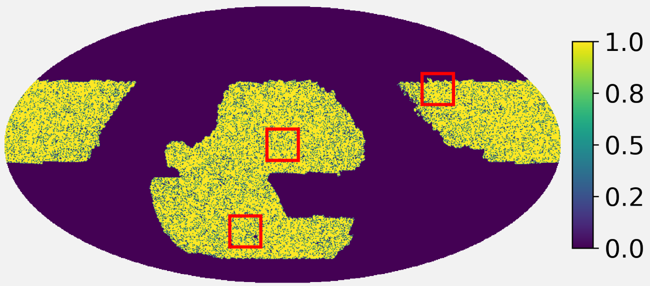
6. Solving the linear equation

$$\hat{\kappa}^{\mathbf{R}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A} + \mathbf{R}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma$$

Convolution kernel matrix	$\mathbf{A}$
Hermitian conjugate of a matrix	$\mathbf{A}^{\mathrm{T}}$
Inverse of a matrix	$\mathbf{A}^{-1}$
Pseudo-inverse of a matrix	$\mathbf{A}^{+}$
Regularization matrix	$\mathbf{R}$

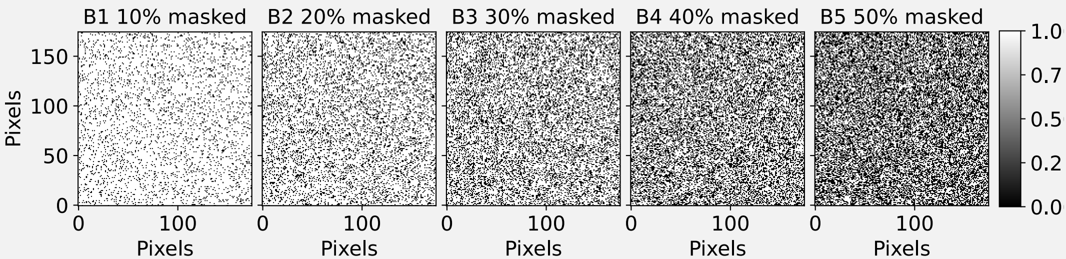
Mask used in simulations

KS will result in a poor estimate of masked regions and near the edge of the footprint



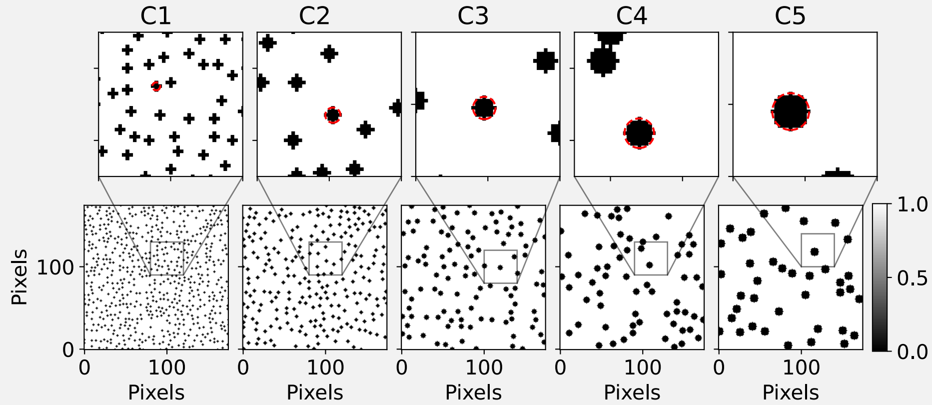
Mask from Observation

The mask is generated from the real observation of the DESI imaging surveys DR8.



Random mask

The masks in panels B1 to B5 have masked pixel rates of 10%, 20%, 30%, 40%, and 50%.



Circular mask

Circular mask with same rate (10%) of masked pixels.

Simulation A: Mask from real observation

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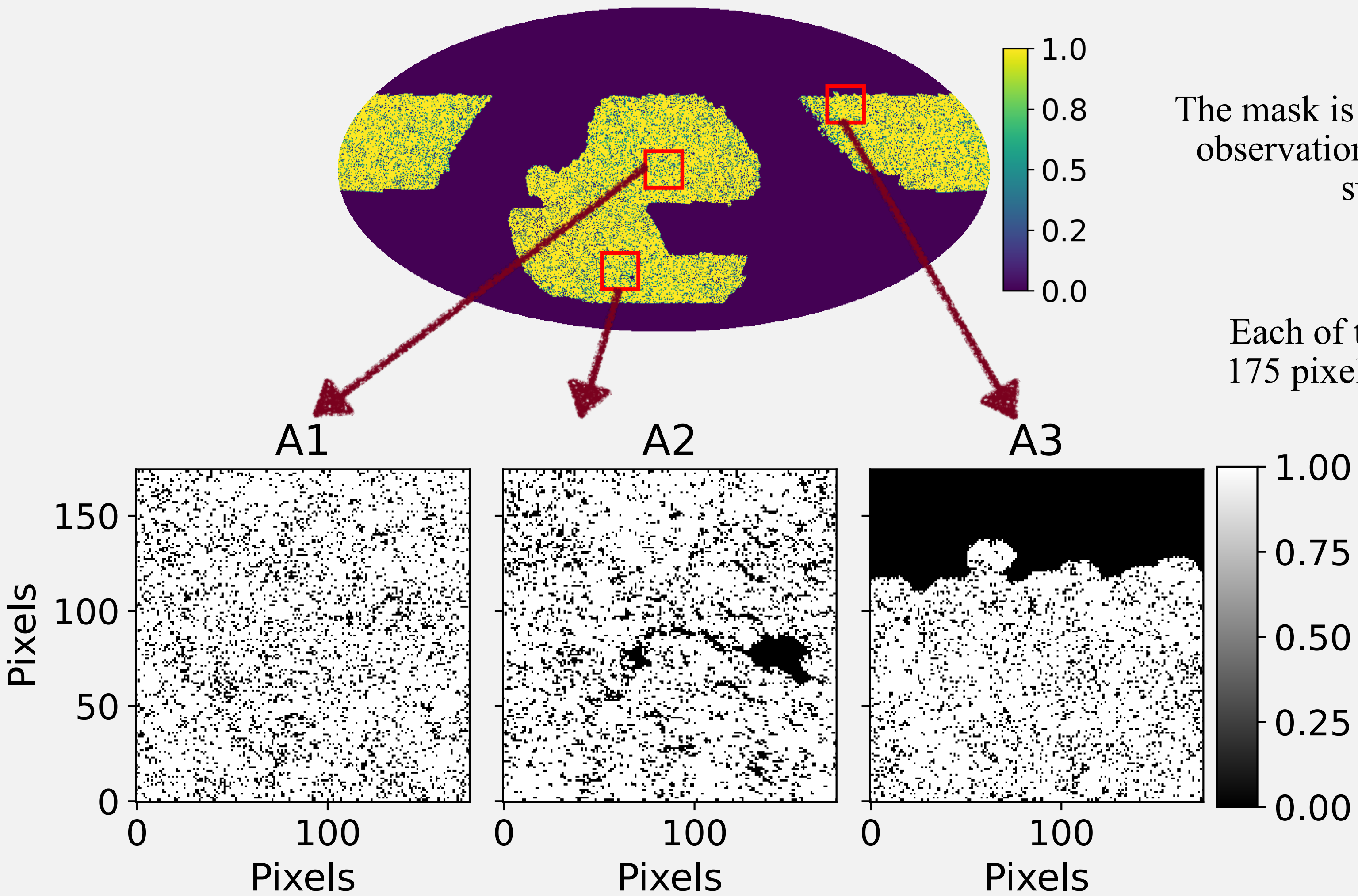
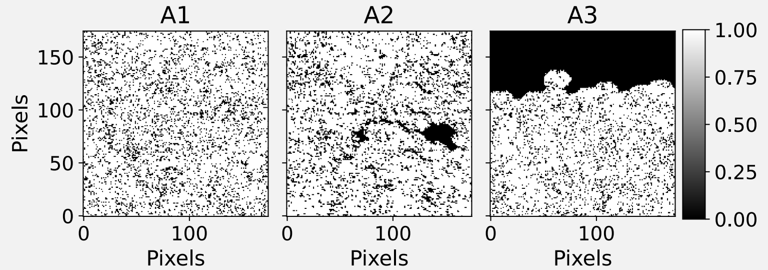


The mask is generated from the real observation of the DESI imaging surveys DR8.

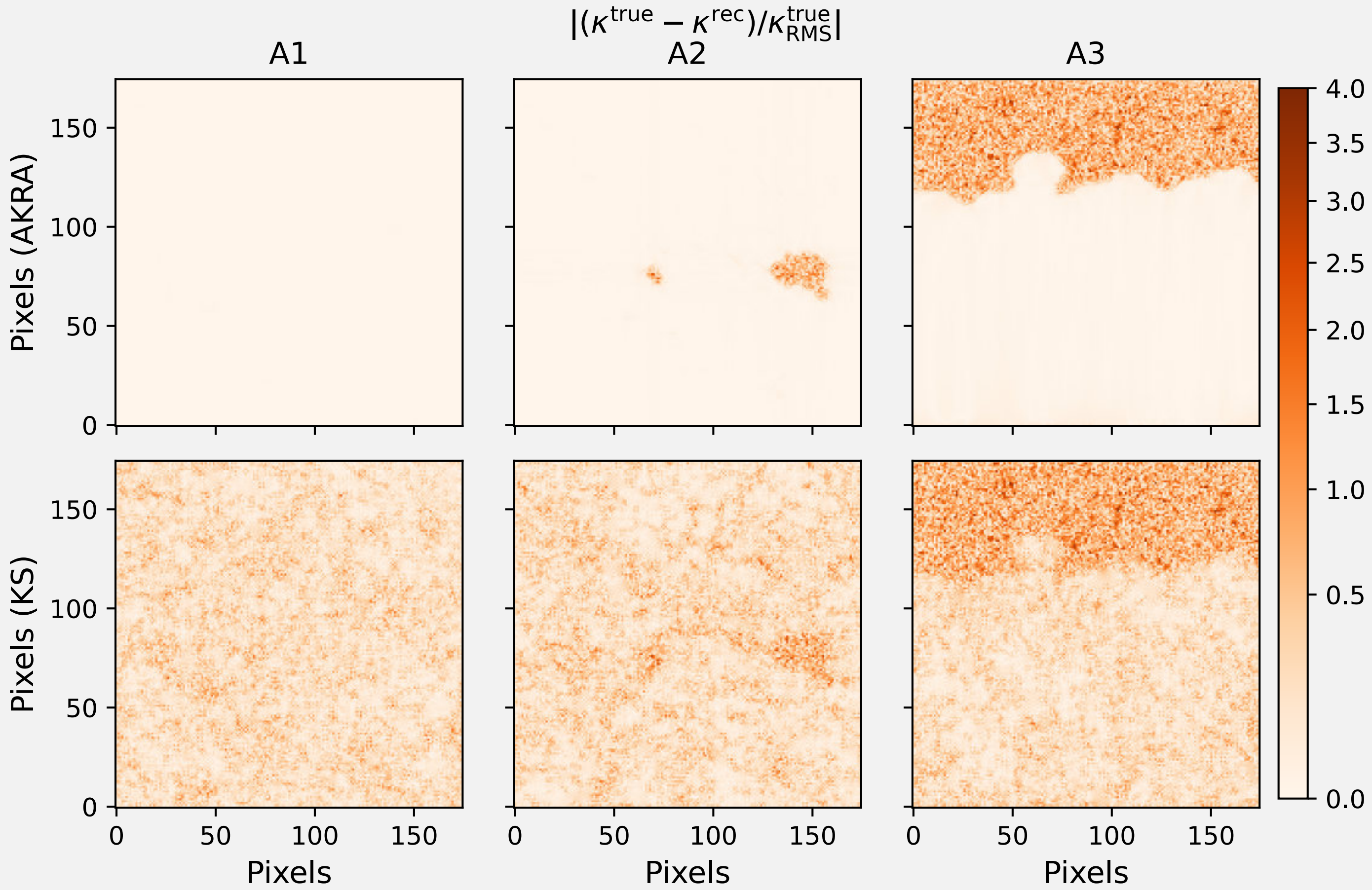
Each of the three patches of sky has 175 pixels, with a side length of 20°.

Three patches of sky with a higher angular resolution of 6.7 arcmin (HEALPix Nside = 512). The **masked pixels** are denoted in **black** and the **unmasked pixels** in **white**.

Simulation 1: Mask from real observation



The residual maps normalized by r.m.s. of the true signal



Reconstructed κ map from AKRA

Reconstructed κ map from KS

Simulation A: results for unmasked pixels

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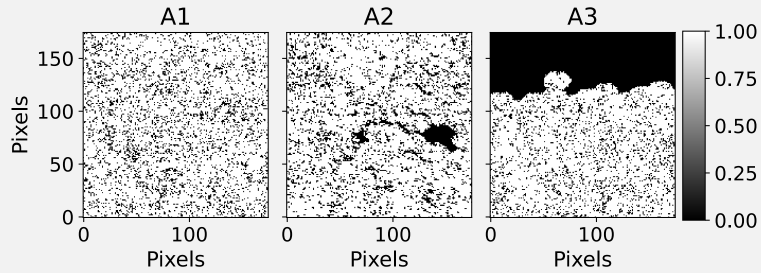
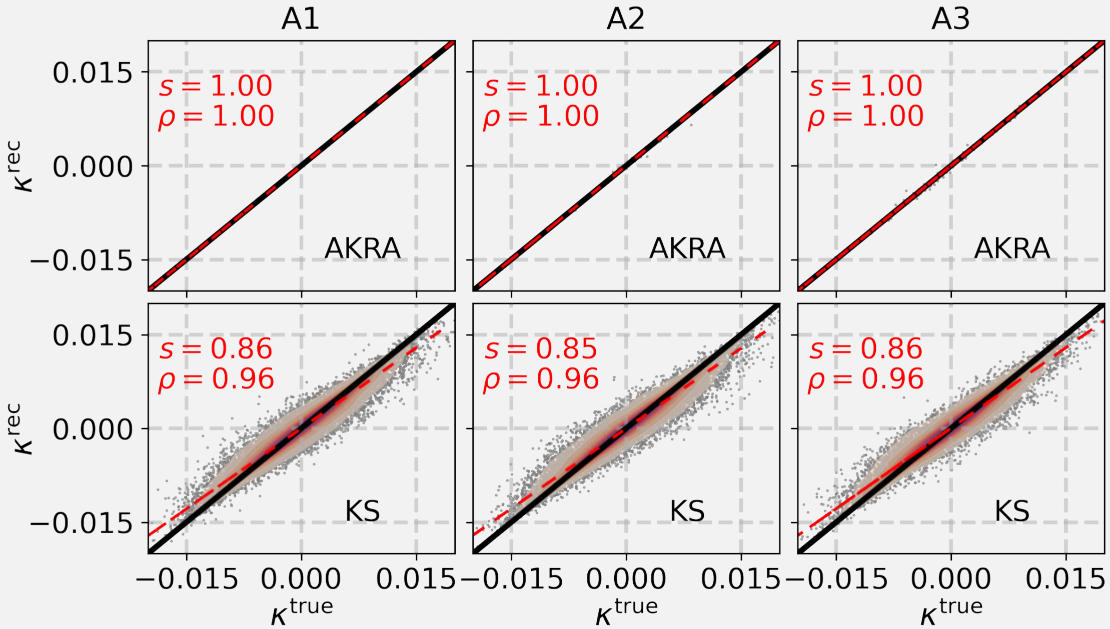
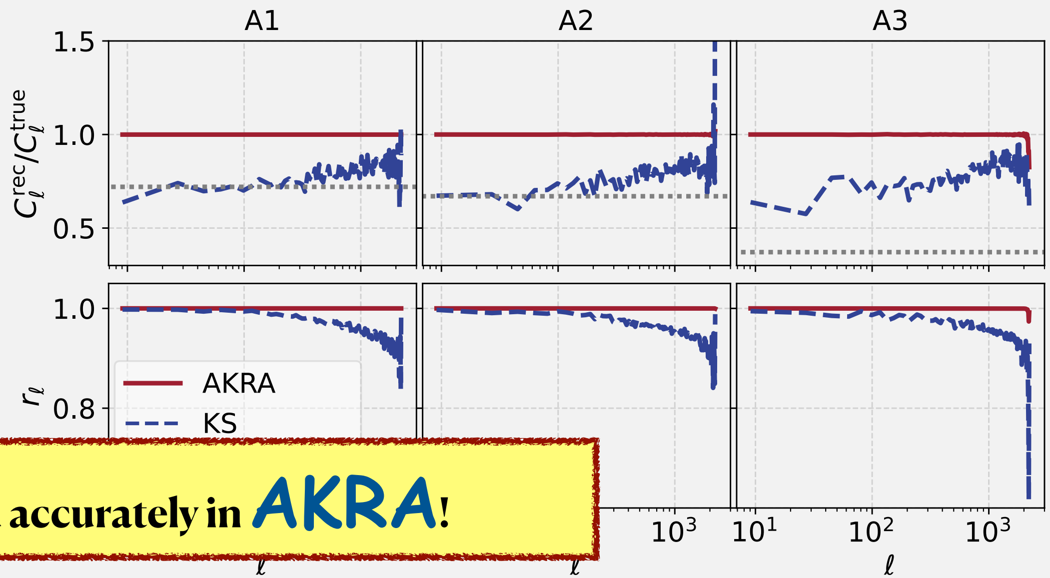






Slop… $\sum \kappa^{\rm rec}\kappa^{\rm true}$

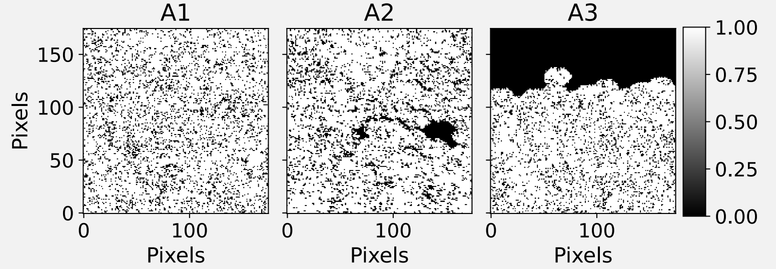
Unmasked pixels (m=1) can be recovered accurately in AKRA!

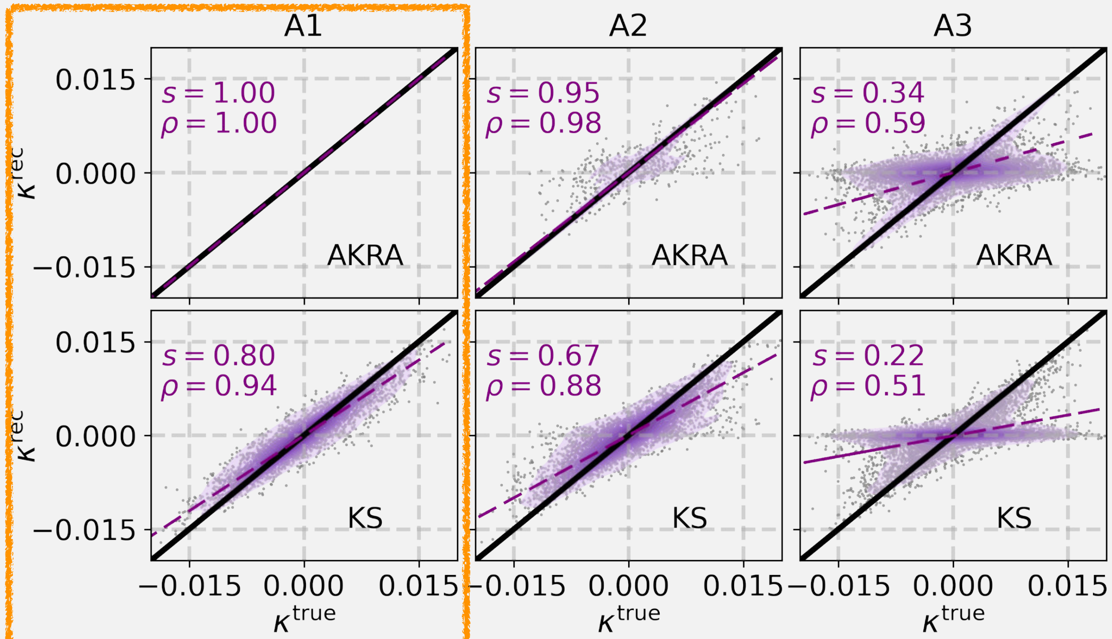
Pearson correlation coefficient (PCC)

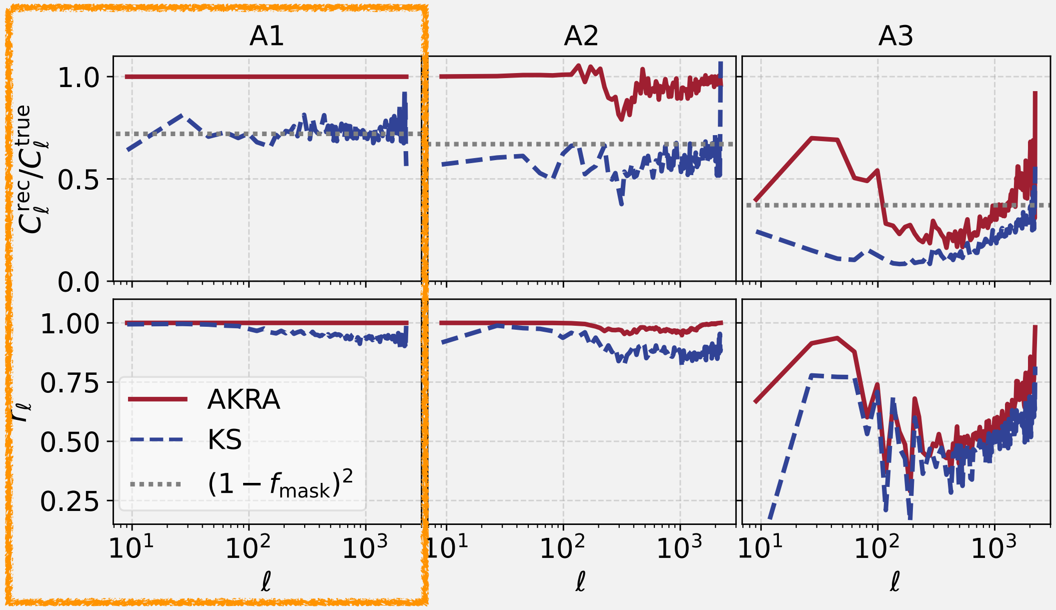
$$\rho = \frac{\langle \kappa^{\rm true}\kappa^{\rm rec}\rangle}{\sqrt{\langle(\kappa^{\rm true})^2\rangle}\sqrt{\langle(\kappa^{\rm rec})^2\rangle}}$$

Cross-correlation coefficient $r_\ell \equiv \frac{C_{\rm rec-true}(\ell)}{\sqrt{C^{\rm rec}(\ell)C^{\rm true}(\ell)}}$

Simulation A: results for masked pixels

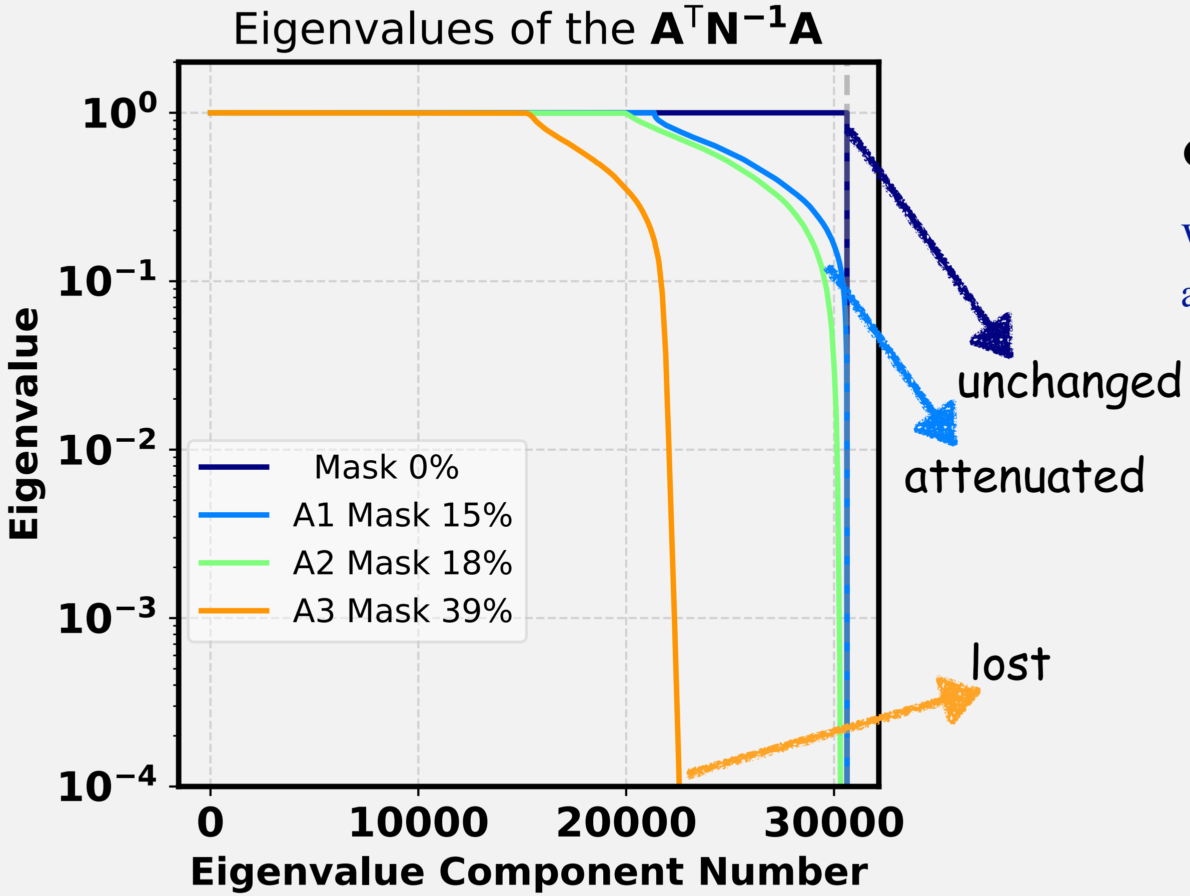






In A1 case, **masked** pixels (m=0) can be recovered accurately in **AKRA**!

Eigenvalues of $\left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}$



$$\hat{\kappa} = \mathbf{D}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma.$$

Ideal case: $\mathbf{D} = (\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A})^{-1}$

$$\mathbf{C} = \mathbf{P}\mathbf{D}^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}$$

Covariance matrix:

We anticipate that the **smallest covariance** will be associated with the **greatest inverse covariance**.

(A1) > (A2) > (A3)

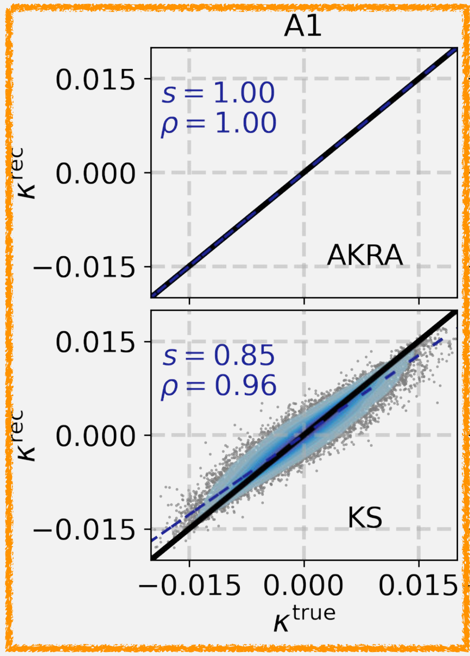
Deconvolution matrix:

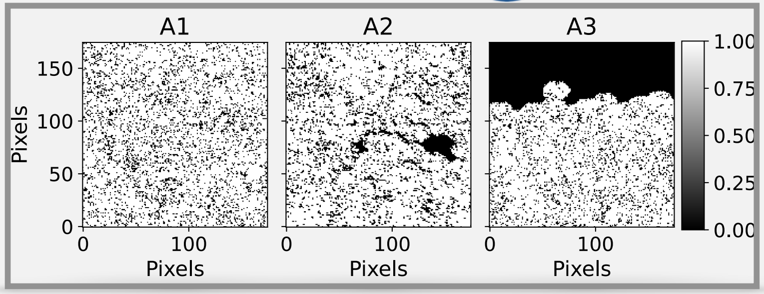
ideal case: $\mathbf{D} = (\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A})^{-1}$

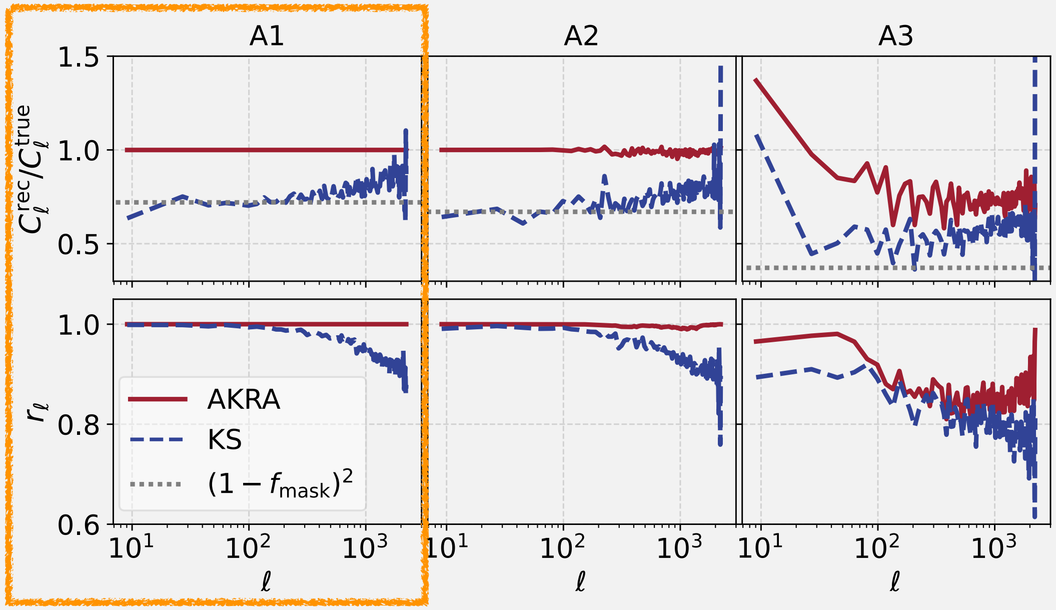
$$\left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{\mathrm{T}}$$

ill-posed case: $\mathbf{D} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A} + \mathbf{R}\right)^{-1}$

Simulation A: results for all pixels

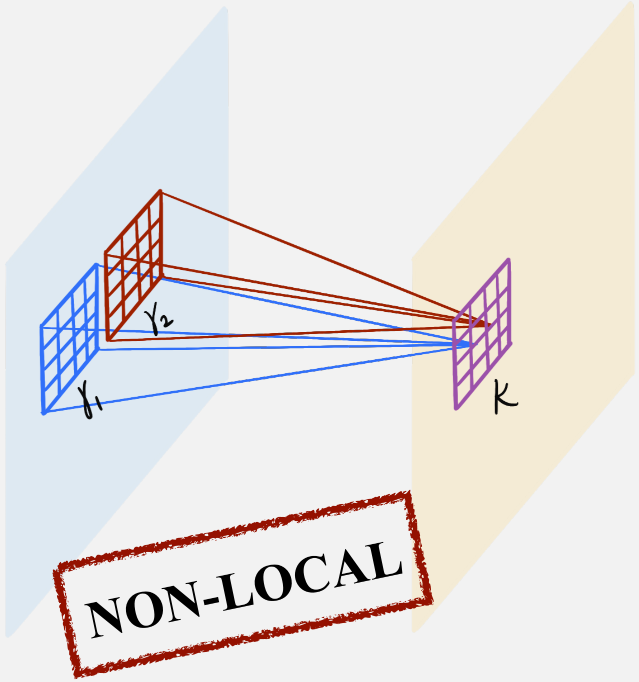






In A1 case, all pixels can be recovered accurately in AKRA!

Conclusion



Why masked pixels can be recovered in AKRA?

Non-local relation between shear and convergence.

- Assumption: flat sky, noise-free, periodic boundary conditions.
- Various mask shapes: C_κ is accurate to 1% or better; $1 - r_\ell \lesssim 1\,\%$ (for masked pixels)
- Future: curved sky, inhomogeneous shape measurement noise ...

Thank you for your time.

https://arxiv.org/abs/2311.00316

[3] arXiv:2311.00316 [pdf, other]

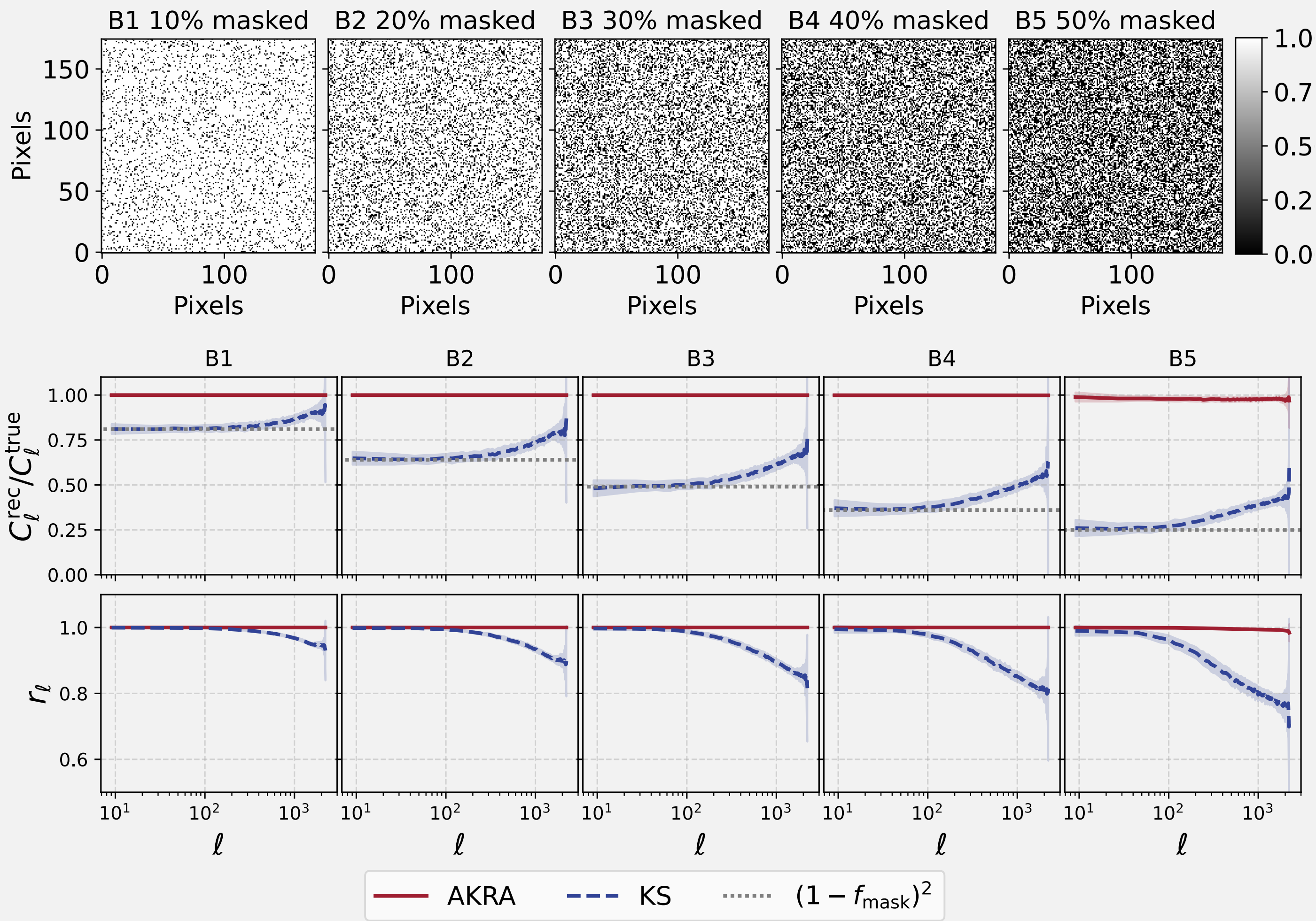
Accurate Kappa Reconstruction Algorithm for masked shear catalog (AKRA)

Yuan Shi, Pengjie Zhang, Zeyang Sun, Yihe Wang

Comments: 17 pages, 21 figures, To be submitted. Comments are welcome!

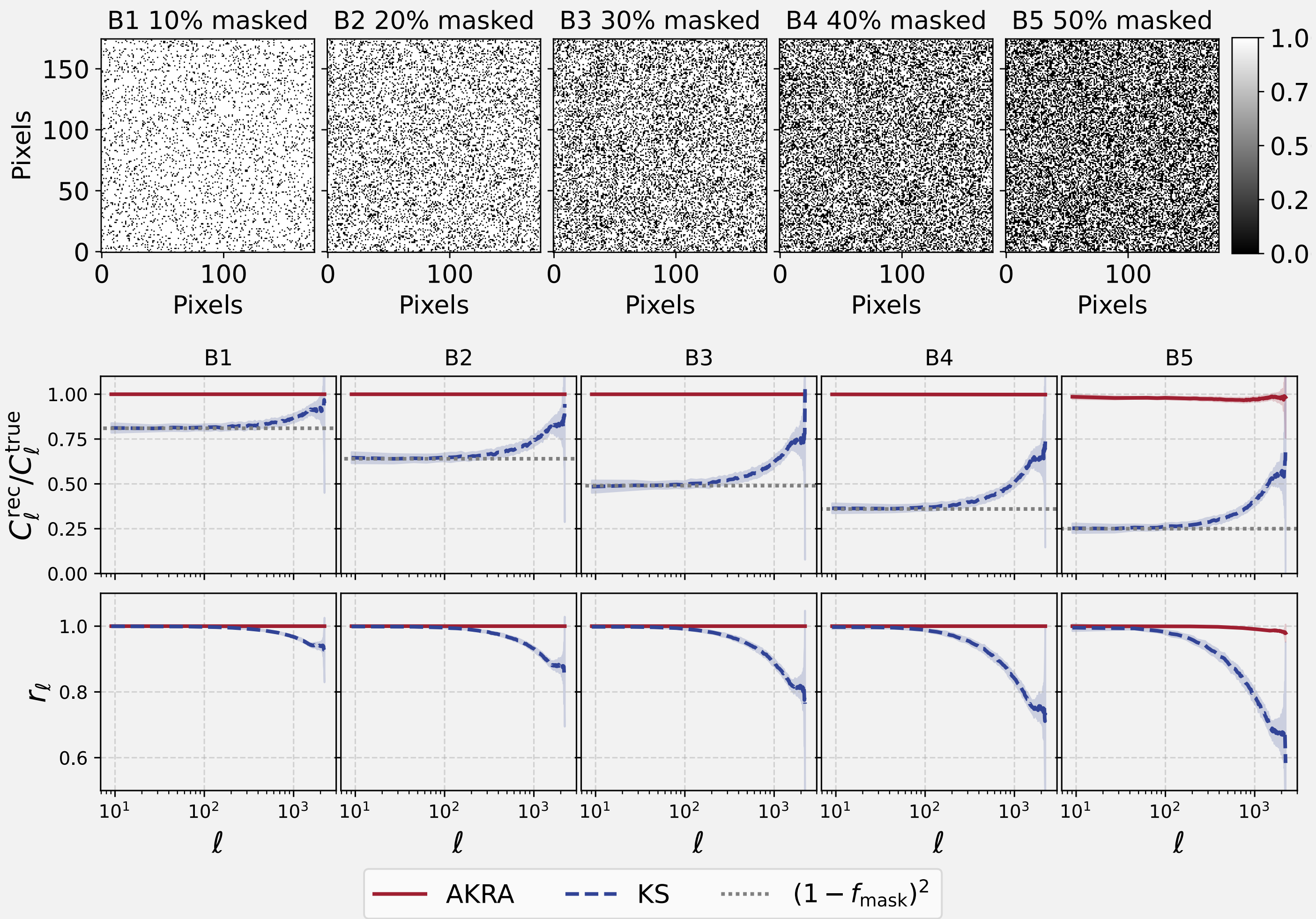
Subjects: **Cosmology and Nongalactic Astrophysics (astro-ph.CO)**; Instrumentation and Methods for Astrophysics (astro-ph.IM)

Simulation B: random mask (results for unmasked pixels)



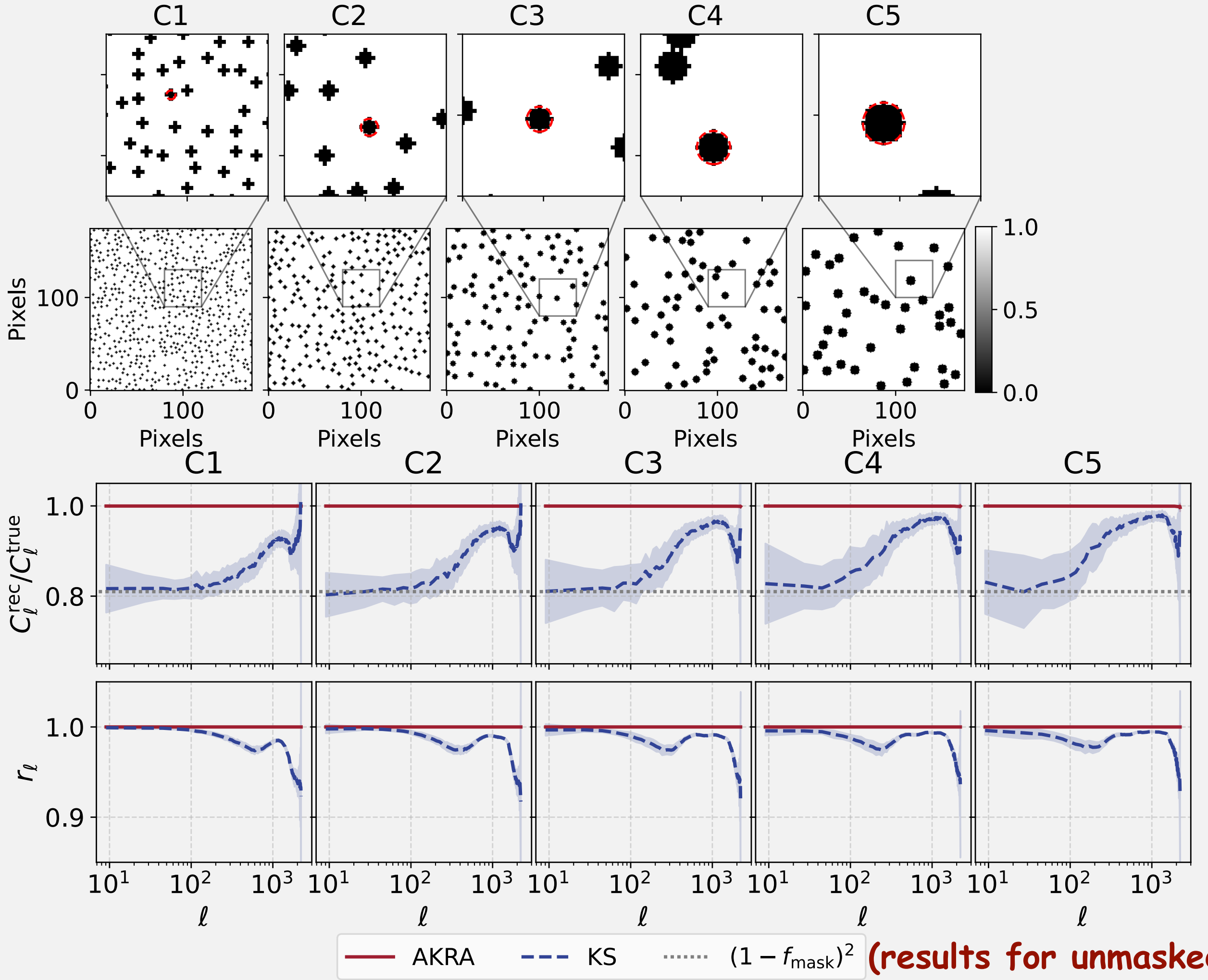
C_κ is accurate to 1% or better;
$1 - r_\ell \lesssim 1\,\%$ (for masked pixels)

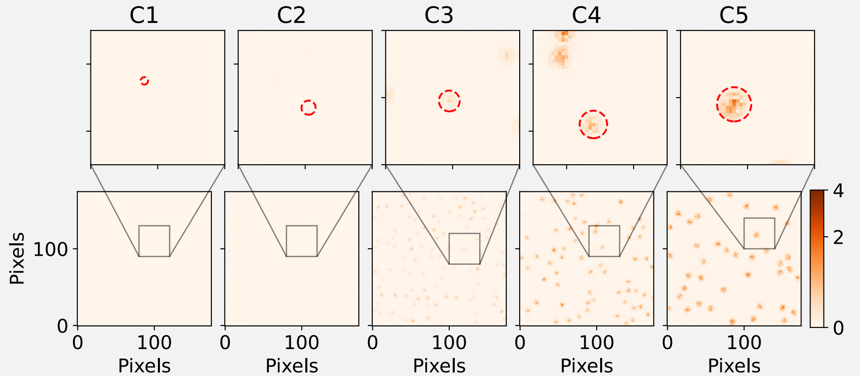
Simulation B: random mask (results for all pixels)

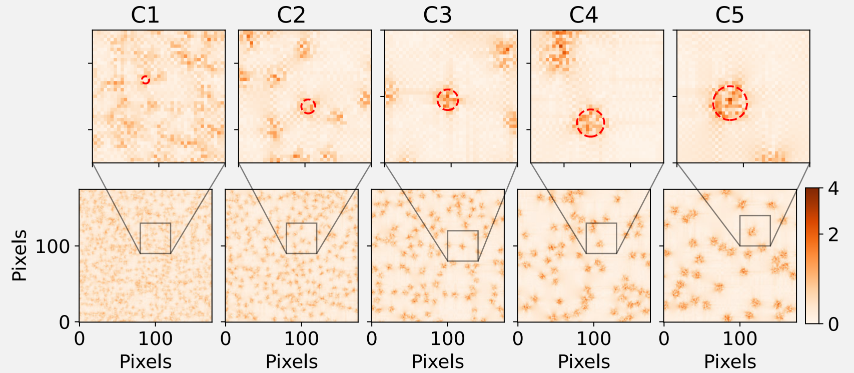


Unmasked pixels can also be recovered in AKAR.

Simulation C: circular mask with same rate (10%) of masked pixels







(results for unmasked pixels)

Eigenvalues of $\left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}$

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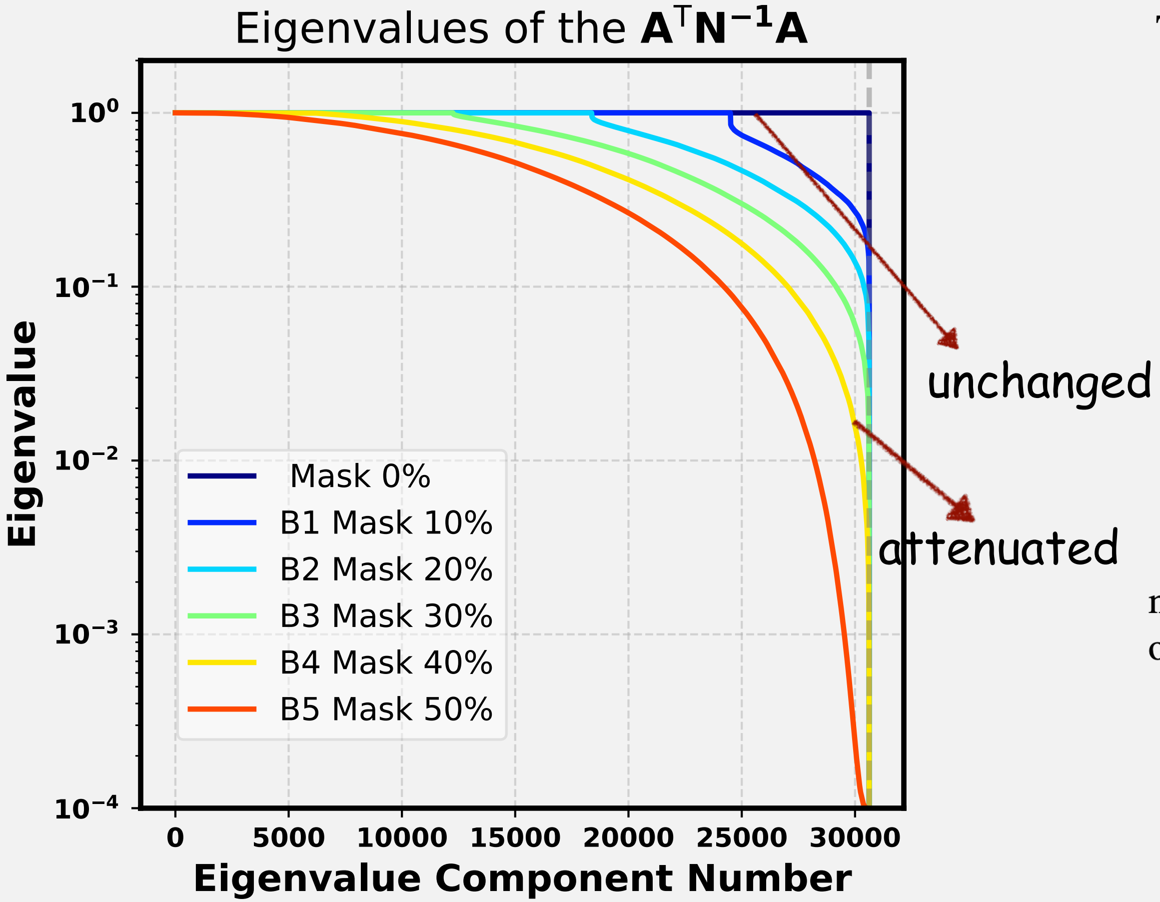


The covariance of the estimator is

$$\Sigma \equiv \left\langle (\hat{\gamma}-\gamma)(\hat{\gamma}-\gamma)^{t} \right\rangle = \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}. \quad (16)$$

We anticipate that the **smallest covariance** will be associated with the **greatest inverse covariance**.

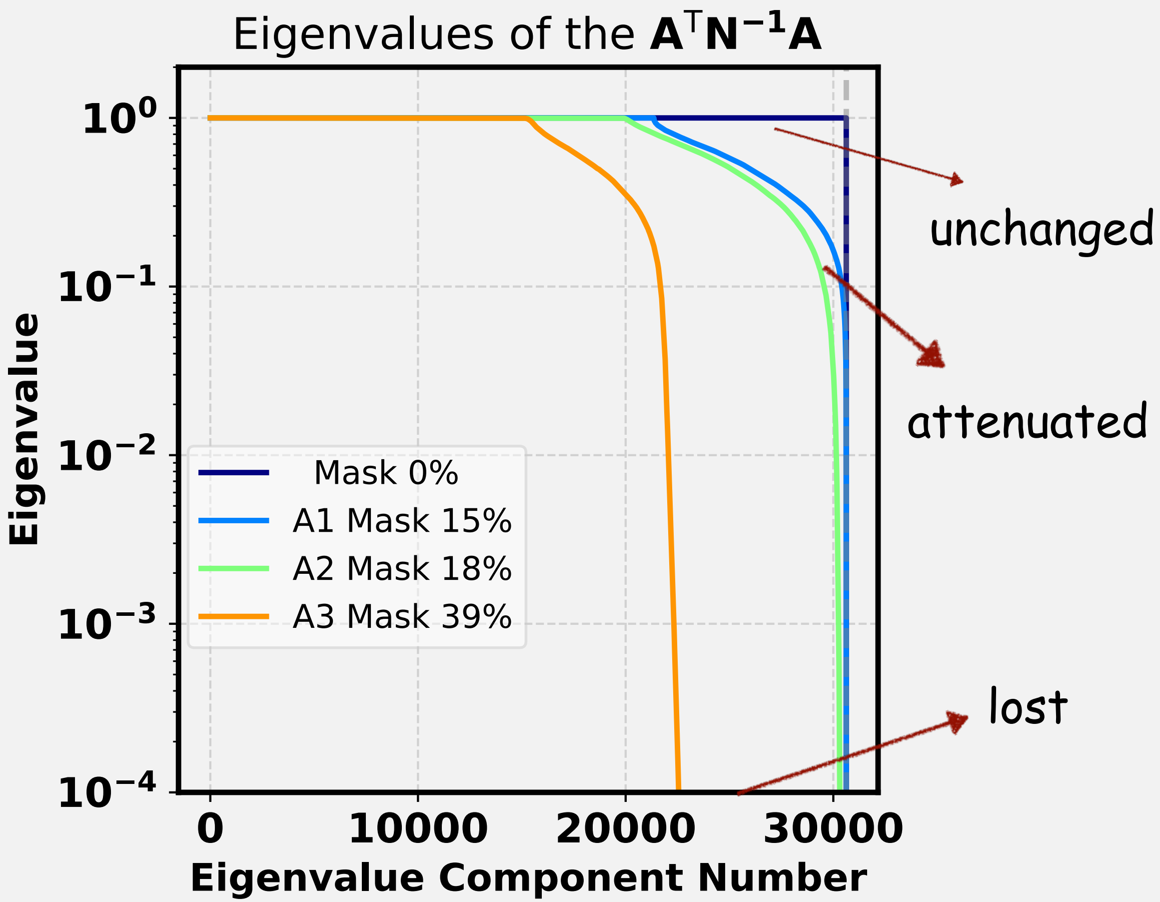
(B1) > (B2) > (B3) > (B4)

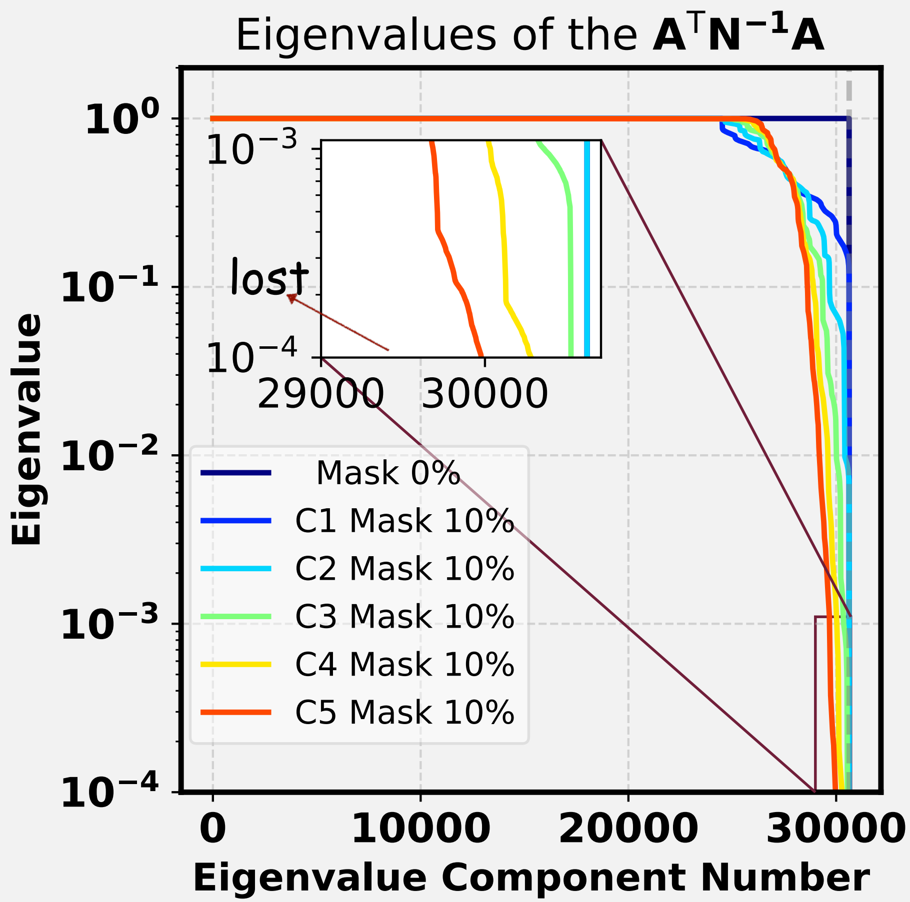
Secondly, the matrix $\left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}$ is the deconvolution matrix of the estimate in Eq. 14. The eigendecomposition of the deconvolution matrix is given by

$$\left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1} = \mathbf{V}\Lambda\mathbf{V}^{\mathrm{T}},$$

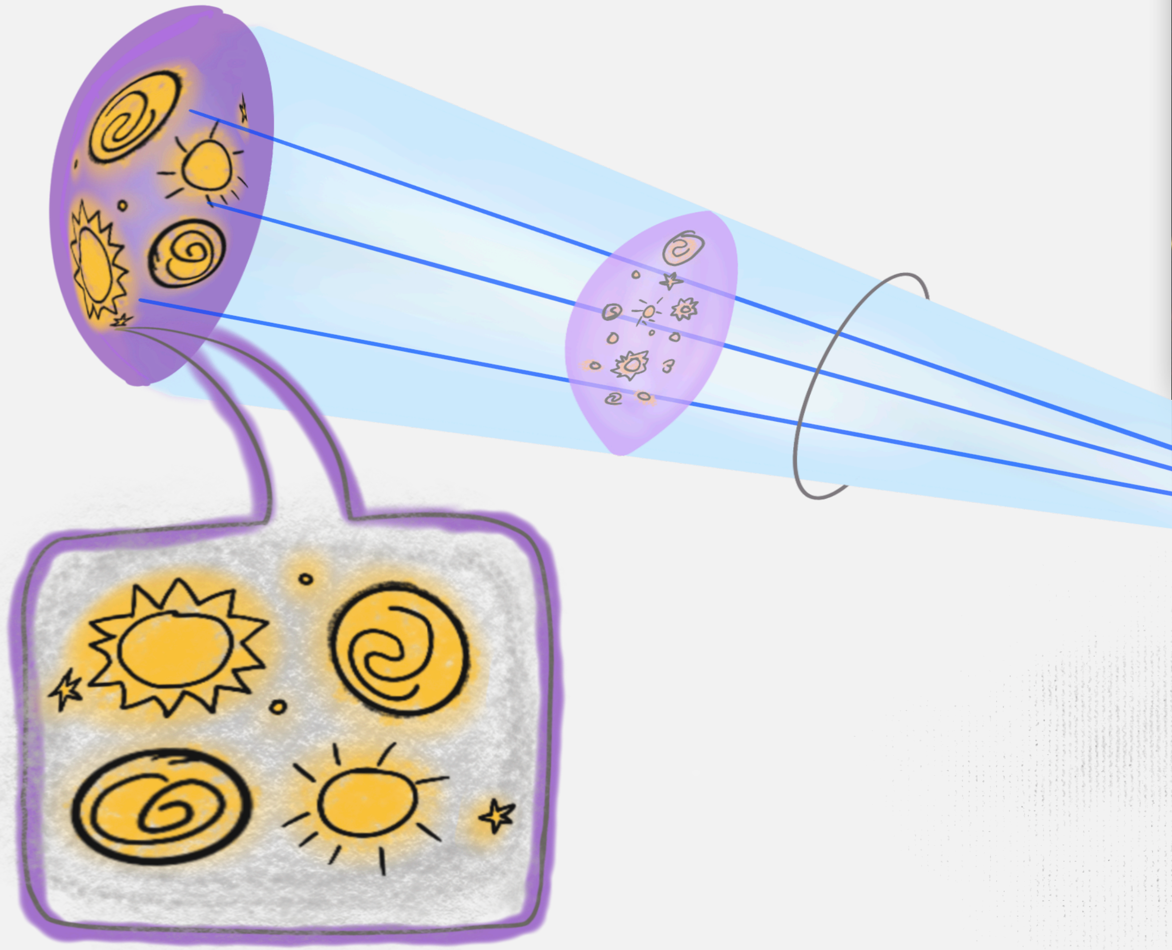
$$\begin{aligned}\hat{\kappa} &= \left(\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma \\ &= \mathbf{D}\mathbf{A}^{\mathrm{T}}\mathbf{N}^{-1}\gamma.\end{aligned} \quad (14)$$

Simulation B: random mask





Gravitational lensing today



Gravitational lensing in the Abell 2218 galaxy cluster, imaged by Hubble Space Telescope.
[Image credit : NASA/ESA]